EXAMPLES IN APPLIED MECHANICS AND ELEMENTARY THEORY OF STRUCTURES

BY

CHARLES E. INGLIS, M.A., A.M.I.C.E.

FELLOW OF KING'S COLLEGE, CAMBRIDGE, AND
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PREFACE

THE following papers in Applied Mechanics and Elementary Theory of Structures are designed for engineering students who have some familiarity with the general principles underlying these subjects. Confidence and facility in applying these principles can only be acquired by assiduous practice in the solution of problems, and for this purpose the student must have ready access to a large stock of miscellaneous examples. The examples contained in text-books are subject to the defect that the principle to be employed in any particular case is of necessity usually indicated by the chapter with which the problem is associated. If his knowledge is to be of any practical value, the student must sooner or later learn to be independent of such extraneous aid, and with the object of helping him towards this state of self reliance, these twenty papers of examples have been compiled. A considerable number of the questions have been taken from examination papers set at the Cambridge Engineering Laboratory, and with the exception of a few more advanced problems in the last five papers the standard agrees with that of the "A" papers in the Engineering Tripos Examination.

C. E. INGLIS.

CAMBRIDGE.
October, 1911.

PAPER No. 1.

1. The framework shewn in Fig. 1 is fixed at B and carried on rollers at A, so that R_A, the reaction at A, is vertical. The framework supports a vertical load of 1000 lbs., and two forces of 400 lbs., due to wind pressure, also act upon the frame. Determine the reaction at B, and construct to scale the reciprocal figure for the framework.

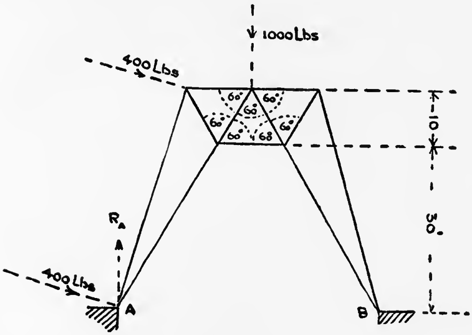


Fig 1.

2. ABC is a continuous rod (*see* Fig. 2) hinged at A. ED is another lever hinged to a fixed point D. The points B and E are connected by a link. A vertical force of 100 lbs. is applied at C, and the lever ED is maintained in a horizontal position by a vertical force P.

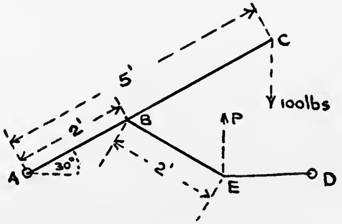


Fig. 2.

Determine

(*a*) The magnitude of the force P,

(*b*) The thrust in the link BE.

Draw to scale the bending moment and shearing force diagrams for the rod ABC.

3. A, A are two parallel rods (*see* Fig. 3) connected together by two transverse bars (B, B). This system slides in fixed guides C, C, and is operated by the eccentric turning about the fixed centre O. The throw of the eccentric is 2 inches, the radius of the circle shewn in the figure is 4 inches, and the eccentric makes one revolution per second. Take the eccentric in the position shewn.

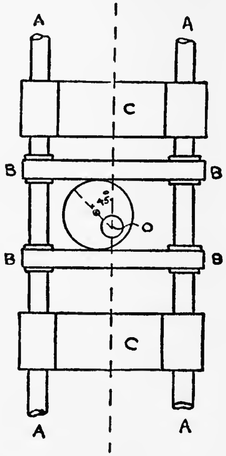


Fig. 3.

Determine

(*a*) The vertical velocity of the moving parts A, B,

(*b*) The relative sliding velocity at the point where the eccentric touches the slide.

(*c*) Suppose the resistance to motion of the AB system is 10 lbs. and the coefficient of friction between the bar B and the eccentric is 0·15. Calculate the torque which is required to rotate the eccentric.

Examine whether the slider can drive the eccentric in this position of the eccentric.

4. Fig. 4 is the velocity-time graph for a given motion. Draw to scale the corresponding acceleration-time graph, and calculate the total distance traversed in the sixty seconds.

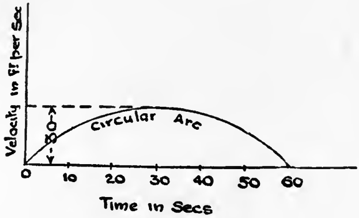


FIG. 4.

The length of the given diagram is to be taken as 6 inches and the height as $1\frac{1}{2}$ inches.

5. A horizontal girder 60 feet long is supported upon two pillars 40 feet apart, one of the pillars being situated at the end of the girder. A wall whose weight is $\frac{1}{2}$ ton per foot run is built over the whole length of the girder.

Draw to scale the bending moment and shearing force diagrams for the girder.

6. The web of the girder is $24'' \times \frac{1}{2}''$.
The flanges are each $9'' \times 1''$.

Calculate for a section 35 feet from the end support

(*a*) The maximum longitudinal stress.

(*b*) The maximum shear stress.

(*c*) The principal stresses at the top and bottom of the web.

7. Take $E = 30{,}000{,}000$ lbs. per square inch.

Calculate

(*a*) The deflection at the outer end of the girder.

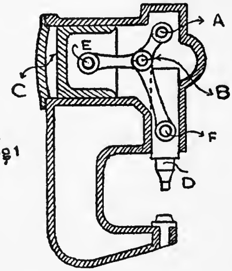
(*b*) The deflection at the point midway between the supports.

8. Draw to scale the deflection diagram for the girder.

PAPER No. 2.

1. Fig. 1 shews the mechanism of a pneumatic riveter. Air under pressure acts upon the piston *C*. The movement of the piston is communicated to the punch *D* through the crank *AB* and the two connecting rods *EB*, *BF*.

Fig. 1



Consider the position in which *AB* makes an angle of 15° with the vertical and *EB* is horizontal, find the velocity ratio of *E* to *D*,

(*a*) Neglecting the obliquity of *BF*.

(*b*) Taking $BF = 2AB$.

(*c*) If the thrust on the piston is 2000 lbs., find for these two cases the thrust exerted by *D*. The efficiency of the machine is to be taken as 75 per cent.

2. A car weighing 1 ton starts from rest on a level road. The tractive force on it is initially 80 lbs. and this falls, the decrease being proportional to the distance travelled, until its value is 30 lbs. at the end of 200 yards, after which it remains constant. There is a constant frictional resistance of 30 lbs. Use the principle of energy to determine the speed at the end of the 200 yards, and plot a curve on a distance basis, shewing the gradual rise of the speed from the start.

3. In a direct acting steam engine the mass of the reciprocating parts may be taken as 1000 lbs. and the mass of the rotating parts as 5000 lbs. The motion of the reciprocating parts may be regarded as a true simple harmonic motion having a travel of 10 inches. The rotating mass may be considered as concentrated at a distance of 2 feet from the axis of the crank shaft. Neglect the effect of friction and use the principle of energy to calculate the percentage fluctuation of speed when the engine is running without steam pressure.

4. When two bodies collide and move on together after the collision, is it the total momentum or is it the total kinetic energy which remains unchanged? Give reasons for your answer.

A railway truck of mass 10 tons moving at a speed of 4 feet per second collides with a similar stationary truck. The collision causes the buffers to compress. The buffer springs are such that the full range of movement of one truck relatively to the other is 9 inches and the reaction between the trucks when the buffers are fully compressed is 5 tons. Calculate whether the buffers will be fully compressed in this case, and if not, find the maximum amount of the compression produced.

5. A steel rod 1″ in diameter is subjected to an axial pull of 10,000 lbs. Its increase of length measured over a distance of 8 inches is found to be ·0033 inch. The same rod subjected to an axial torque of 100 foot-lbs. is found to twist through an angle of 27·6 minutes measured over this same distance. From these observations deduce the values of E, C, and m for the material.

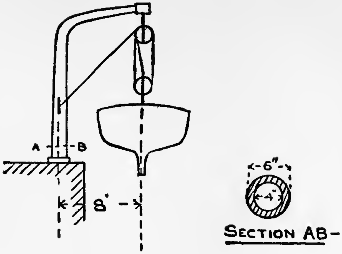


Fig 6

6. The arrangement of a ship's davit is shown in Fig. 6. The vertical load carried by the davit is 2 tons. Calculate the greatest compression and tension stresses set up at the cross-section *AB*.

7. If a prism is free from surface forces over a portion of its length, shew that for a cross-section taken in this region the shear stress at the boundary has its direction along the boundary.

A straight horizontal tube 40 inches long, 1 inch external diameter, $\frac{1}{50}$ inch thick, rests upon end supports and carries a load of 50 lbs. at the centre.

(*a*) Calculate the greatest longitudinal stress set up in the material.

(*b*) Calculate for a section 10 inches from one end the greatest shear stress.

(*c*) Shew by means of a graph drawn to scale the variation in the shear stress taken along the section from the lowest point to the highest.

8. A three-hinged arch, having the hinges at the springings and the crown, is parabolic in form, the span being 100 feet and the rise 25 feet. It carries a load of 50 tons distributed uniformly (horizontally) over the right-hand half span. Calculate the horizontal thrust at the springings and draw to scale the bending moment for the two halves of the arch.

PAPER No. 3.

1. The C.G. of a locomotive is 6 feet above rail level.

The centres of the rails are 5 feet apart.

A curve of 8 chain radius is built with a super-elevation designed for a speed of 20 miles per hour. Find how fast the locomotive can take the curve without overturning.

2. A four-wheeled railway truck is rounding a curve at a speed of 20 miles per hour. The flanges of the wheels *A* and *D* are rubbing against the rails, the points of contact having the positions *P* and *Q* shewn in Fig 1.

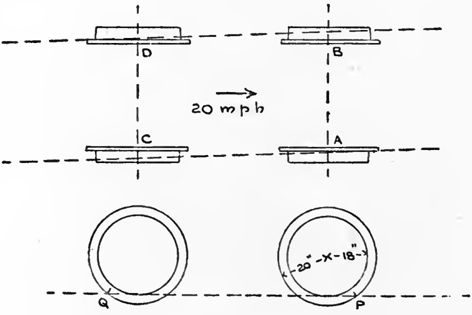


Fig 1

If the thrust between the flanges and rail is in each case 400 lbs., and the coefficient of friction is 0·2, calculate the drawbar pull required to maintain the motion.

3. A drawbridge *AB* (see Fig. 2) is counterbalanced by a weight *E* which is connected to the outer end of the bridge by a rope *ECB* passing over a pulley wheel at *C*. The weight descends along a curved incline and the shape of this incline is such that a balance is obtained for all positions of the bridge.

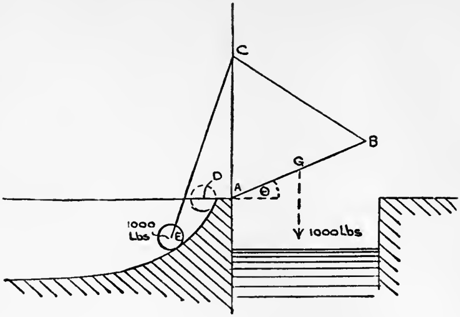


Fig 2.

When AB is horizontal, the counterweight has its centre at D. Use the principle of work to determine the position of the counterweight for the following positions of AB, $\theta = 15°, 30°, 45°, 60°, 75°, 90°$. $AB = AC$. $AG = GB = 15'$. $AD = 4'$.

4. A rod is guided so as to lift up and down freely in a vertical line. A wedge whose angle is α rests on a smooth horizontal surface and its thin edge is inserted beneath the lower extremity of the vertical rod. By means of a thrust, applied horizontally to the wedge, the lower end of the rod is made to rise in contact with the wedge along a line of greatest slope. Considering this arrangement as a machine for lifting the rod and taking ϕ as the angle of friction between the rod and wedge, calculate

(*a*) The mechanical advantage of the system.

(*b*) Its efficiency.

5. Fig. 3 illustrates a small foundry crane which is capable of rotation about the vertical axis AB.

Draw the B.M. diagram for the vertical and horizontal girders.

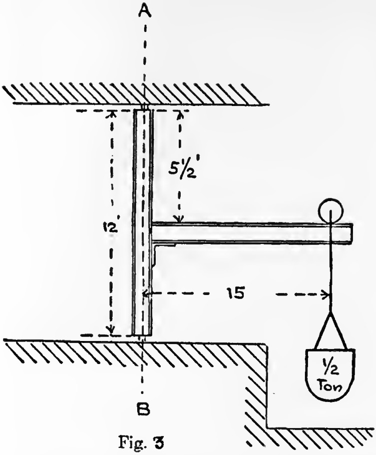


Fig. 3

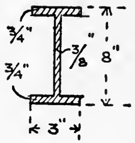


Fig 3a

If the section of the vertical member is as shewn in Fig. 3*a*, determine the greatest longitudinal stress in the material at a section 5 feet from the top.

6. Fig. 4 represents a beam strengthened by tie bars and vertical struts. By tightening the tie bars and consequently increasing the thrusts exerted by the struts, it is possible to arrange that the beam shall be free from bending moment at the sections over the struts.

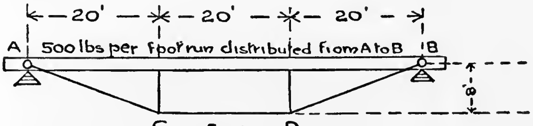


Fig 4

Draw carefully to scale the bending moment diagram for this state of affairs and scale off the greatest bending moment to which the beam is subjected.

Calculate also the tension in the horizontal tie bar.

7 A straight prism is subjected to a longitudinal compression stress of 20,000 lbs. per square inch. Calculate the thrust per square inch which must be laterally applied in order to prevent the prism expanding laterally. Determine also the magnitude of the longitudinal stretch

$$E = 30{,}000{,}000 \text{ lbs. per square inch.}$$

$$m = \frac{10}{3}.$$

8. A propeller shaft has a diameter of 12 inches.

(*a*) Calculate the greatest twisting couple it can withstand if the maximum stress difference in the material is not to exceed 10 tons per square inch.

(*b*) Calculate also the twist in the shaft per foot run. C for the material is 12,000,000 lbs. per square inch.

NOTE. The stress difference means the difference between the greatest and least principal stresses at a point in the material.

PAPER No. 4.

1. A body of mass M is moving in two dimensions. The angular velocity of the body is ω and the velocity of its C.G. is v. Shew that the kinetic energy of the body is $\frac{1}{2}Mv^2 + \frac{1}{2}Mk^2\omega^2$, where k is the radius of gyration of the body with reference to an axis through its C.G. perpendicular to the plane of motion. A wheel is fitted with an axle of radius a, which projects on each side of the wheel. The axle rolls on two parallel rails, each inclined at an angle α to the horizontal, so that the wheel moves in a vertical plane between the rails. Shew by the principle of energy that, when the centre of the wheel has moved through a distance s from rest, the velocity is given by

$$v^2 = 2\frac{a^2}{a^2 + k^2} gs \sin\alpha.$$

2. A fly-wheel weighing 100 lbs. is in the form of a uniform disc 18 inches in diameter and is mounted upon a spindle one inch in diameter. The wheel rotates without appreciable friction and winds up a weight by a string wound round the spindle. It is observed that the speed of the wheel falls from 120 to 105 revolutions per minute in 35 seconds. Determine the weight which is being wound up.

3. Fig. 1 represents a centrifugal governor.

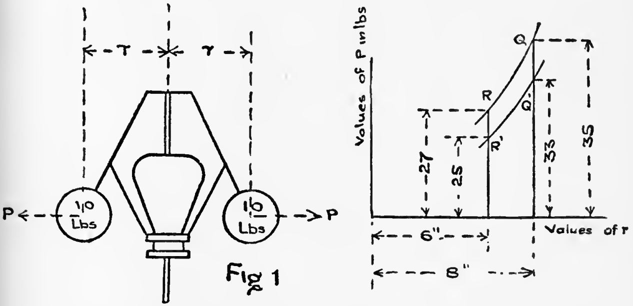


Fig 1

When the governor is at rest experiments are performed to find the magnitude of the horizontal force P required to move the balls for various values of r. The results of these experiments are given by the curves RQ, $R'Q'$. RQ gives the force required to pull the balls outwards, $R'Q'$ gives the force required to prevent them moving inwards.

When $r = 6''$ the throttle valve is fully open.
„ $r = 8''$ „ „ „ entirely closed.

Calculate from the above data the limits of speed, stated in revolutions per minute, between which it is possible for the governor to run.

4. Fig. 2 shews a construction of square section bars and round section rods, hung over a knife edge. The bars and rods

are made of the same material. Calculate the periodicity for small oscillations of the system about the knife edge.

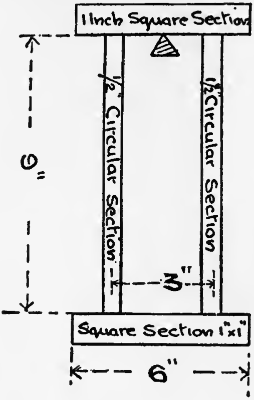


Fig 2

5. A straight steel rod 1 inch in diameter is subjected to bending by means of terminal couples. If the elastic limit is reached when the longitudinal stretch is $\frac{1}{800}$, calculate the maximum curvature which can be given to the rod without producing permanent set. Determine also the magnitude of the corresponding terminal couples. E for the steel $= 30 \times 10^6$ lbs. per square inch.

6. A helical spring of small pitch angle is made out of steel wire $\frac{1}{4}$ inch in diameter. The centre line of the wire lies on a cylinder 3 inches in diameter and there are 100 complete turns. The spring is hung up by one end and to the other end is attached a weight of 20 lbs. Calculate the greatest shear stress set up in the material and find the axial elongation produced in the spring. C for the material is 12,000,000 lbs. per square inch.

7. Two loads of 10 and 20 tons respectively separated by a distance of 8 feet advance along a horizontal girder 20 feet long

supported at its extremities. Determine the position of the section which is subjected to the greatest bending moment and calculate the magnitude of this bending moment.

8. Fig. 3 is the plan view of a horizontal shaft AB 2 inches in diameter encastered at the end A and carrying a horizontal cross piece BC with a weight of 100 lbs. at C. Consider the section PQ 10 inches from A and determine the principal stresses at the top and bottom points of this section.

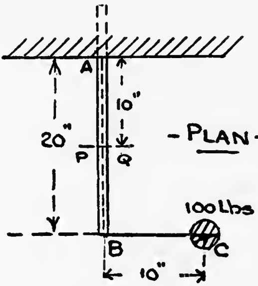


Fig 3

PAPER No. 5.

1. The mechanism of a platform weighing machine is shewn in Fig. 1.

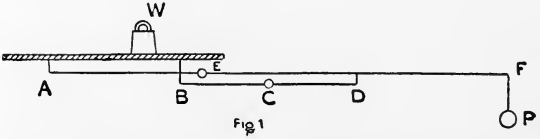


Fig 1

AEF is a lever pivoted at *E*, *BCD* is a lever pivoted at *C*. The platform bears directly on the ends *A* and *B* of the two levers by means of the short struts shewn in the diagram and the end *D* of the lever *BCD* presses upwards against the lever *AEF* at the middle point of *EF*.

The load W is balanced by a weight P carried at F. Shew that if $BC = CD$ and $EF = 2EA$ then $P = \frac{W}{2}$ for all positions of W on the platform.

2. A spiral spring is hung up vertically by one end and to the other end is attached a weight of 20 lbs. The spring is stretched thereby through a distance of 3 inches. If the weight is made to perform vertical oscillations about its position of statical equilibrium, calculate the number of complete oscillations which occur in one minute. How would the number be affected by the addition of 10 lbs. to the existing weight?

3. Consider the spiral spring and 20 lb. weight mentioned in question 2, and suppose that the weight is oscillating 2 inches above and below its position of statical equilibrium. Shew by means of curves drawn to scale, the variation of (1) the potential energy of the mass, (2) the kinetic energy of the mass, (3) the strain energy of the spring, as the mass moves from its lowest to its highest position.

4. A horizontal girder of I section 20 feet long rests upon end supports. A weight of 20 tons is concentrated at the middle point of the girder.

The flanges of the girder are $12'' \times 1''$, and the web is $20'' \times \frac{1}{2}''$.

The value of E for the material is 30,000,000 lbs. per square inch. Neglect the mass of the girder and determine the number of oscillations made per second when the vertical load is slightly displaced from its position of statical equilibrium.

5. A flanged pipe joint is made with 6 steel bolts and elastic packing. The bolts are screwed up so that the initial load on each is 2 tons, and the compression in the packing is twice as great as the extension in the bolts. When the water pressure comes on the pipes the flanges are pulled apart with a force of 10 tons. Shew that the total load on each bolt is 3·11 tons.

6. If a is the radius of a helical spring of small slope, r the radius of the wire composing it, l the length of the wire in the spring and P the axial load; calculate the axial elongation and shew thence that the energy stored in the spring is $\frac{P^2a^2l}{\pi Cr^4}$ where C is the modulus of rigidity.

7. In Fig. 2 the vertical rod CD is encastered at C, its lower end fits tightly into the horizontal bar AB. Take E for the rod and bar as 30,000,000 lbs. per square inch and compute the vertical and horizontal displacements of the point B.

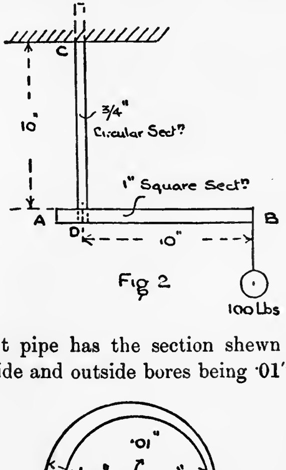


Fig 2

8. A straight pipe has the section shewn in Fig. 3, the centres of the inside and outside bores being ·01″ apart.

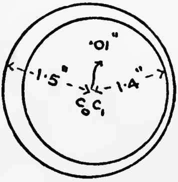
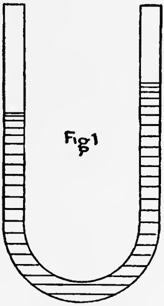


Fig 3

The tube is subjected to a pull of 2 tons applied along the axis of the outside of the tube; determine the greatest and least longitudinal stress set up in the material.

PAPER No. 6.

1. Fig. 1 illustrates a U tube placed with its arms vertical and partially filled with liquid. If a small difference in level of the liquid in the two arms is established, shew that the time of oscillation of the level about its mean position is the same as that of a pendulum of length $\frac{l}{2}$ where l is the length of tube occupied by the liquid.



2. A rope is coiled round two fixed bollards in the manner shewn in Fig. (2) and one end is held with a force of 40 lbs. Calculate the greatest force which can be applied at the other end without causing the rope to slip. The coefficient of friction between rope and bollard may be taken as 0·3.

3. A plate of area A is immersed with its plane vertical in a liquid of density w. The depth of the C.G. of the area is d. Prove that the total pressure on the area is wAd.

If Ak^2 is the moment of inertia of the area about a line in which the plane of the area intersects the surface, prove that the depth of the centre of pressure is $\frac{k^2}{d}$.

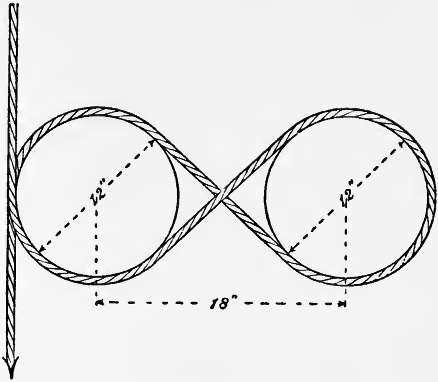


Fig. 2.

If the plate is inclined at an angle θ to the vertical, how are these results affected?

Find the depth of the centre of pressure of a circular plate 1 foot in diameter immersed with its plane vertical and its centre at a depth of 3 feet below the surface.

4. A cylindrical drum 10 feet in diameter, capable of turning freely about a horizontal axis, has wound upon its surface a wire rope 150 feet long. One extremity of the rope is attached to the surface of the drum, while the other end after hanging vertically for a short distance carries at its extremity a load of 3 tons.

The mass of the drum is 2 tons, its radius of gyration is $4\frac{1}{2}$ feet and the wire rope weighs 3 lbs. per foot run. The system starts from rest. Calculate the angular velocity of the drum when the weight has fallen a distance of 120 feet.

5. The shaft of a Laval steam turbine is transmitting 10 horse-power at 30,000 revs. per minute. If the shear stress is not to exceed 6 tons per square inch, calculate the least possible diameter for the shaft.

6. The coupling rod for a six feet four-coupled locomotive is of *I* section, flanges $\frac{7}{8}'' \times 2\frac{3}{4}''$, web $3\frac{3}{4}'' \times \frac{3}{4}''$.

The length between centres is 9 feet and the rod is operated by a crank of 13 inches throw. If the engine runs at a speed of 75 m. p. h., calculate the greatest longitudinal stress set up in the material owing to the mass and weight of the coupling rod.

The weight of the material is 480 lbs. per cu. foot.

7. A straight vertical column is encastered at its lower end, the upper end being quite free. Determine an expression for the greatest vertical load which can be applied at the upper end without causing failure in the strut.

8. The web of a girder of *I* section is 6 feet deep and $\frac{1}{2}$ inch thick. The girder is supported upon the ground along its whole length, the web being vertical. The web is devoid of stiffeners. Calculate the greatest load per foot run which can be applied to the upper flange without producing instability in the web.

(*a*) Allowing the top flange entire freedom.

(*b*) If the top and bottom flanges remain parallel and vertically above one another.

E for the material in 30×10^6 lbs. per square inch.

PAPER No. 7.

1. Fig. 1 represents the form of steering gear known as a Rapson's Slide. The slide S is constrained to move along the straight line AB. Assuming that the coefficient of friction between slide and tiller is ·08, calculate the pull which must be applied to the wire rope to move the tiller to the left in the position shewn, the couple to be overcome amounting to 40 ons-feet.

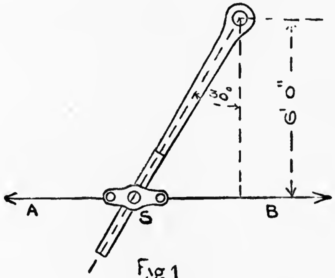


Fig 1

2. A ship of 1000 tons is towing a vessel of 500 tons by means of a steel hawser, 300 feet long. The hawser is allowed to become slack and just before it becomes taut again the speeds of the vessels are 6·5 and 5 feet per sec. respectively. Calculate the speed common to the two vessels when the hawser once more becomes taut and determine the consequent loss of kinetic energy.

Assuming that this loss of kinetic energy is all accounted for by strain energy in the hawser, calculate the intensity of the tensile stress set up.

The sectional area of the hawser is 6 square inches and the value of E for the material is 14000 tons per square inch.

3. A straight plank of uniform section has its upper end in contact with a vertical wall and its lower end in contact with the ground. The plank is sliding down, the motion occurring in a vertical plane. In the initial position the plank was inclined at an angle α to the vertical. Write down the equations of motion assuming no friction and shew that the upper end remains in contact with the wall until $\cos\theta = \frac{2}{3}\cos\alpha$.

4. The arrangement of a flap-valve closing the delivery end of a pipe is shewn in Fig. 2. Calculate the difference of water level which can be maintained by this arrangement.

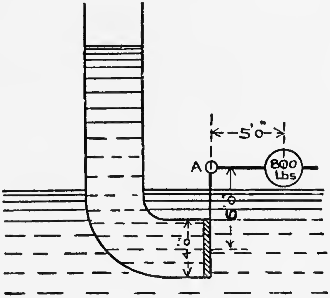


Fig 2

5. Fig. 3 represents a cantilever hydraulic crane. The single rope which supports the load is led over two frictionless pulleys and then vertically down the axis of the girder to the hydraulic apparatus. Consider the section AB and determine the greatest tensile and compression stress across the section.

6. Shew that m for an elastic material cannot have a value lying between -1 and $+2$.

A sphere of steel 1 foot in diameter is subjected to a hydrostatic thrust of 10 tons per square inch. Calculate the decrease in volume measured in cubic inches.

$$E = 30 \times 10^6, \quad \text{and } m = \tfrac{10}{3}.$$

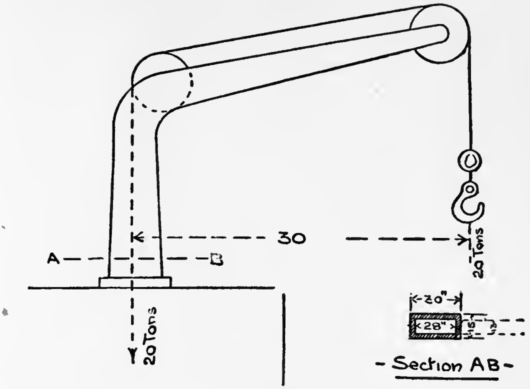


Fig. 3.

7. A long straight tube 2 inches in diameter and $\frac{1}{10}$ inch thick has its ends closed and is subjected to an internal water pressure of 1000 lbs. per square inch. Calculate the normal stresses over a longitudinal and a circumferential seam.

If in addition the tube is subjected to a torque of 500 foot-lbs., calculate the greatest stress difference and the greatest stretch in the material.

$$E = 30 \times 10^6 \text{ lbs. per square inch,} \quad \text{and } m = \tfrac{10}{3}.$$

8. The hub of a wheel is connected to the rim by 20 straight radial spokes encastered at both ends. The spokes are rods of circular section $\frac{1}{2}$ inch in diameter and their free length is 20 inches.

Calculate the force required to displace the hub relatively to the rim through a distance of $\frac{1}{10}$ inch along the axis of the wheel. $E = 30 \times 10^6$ lbs. per square inch.

PAPER No. 8.

1. A steel wire is stretched between two points at the same level 60 yards apart. The material of the wire weighs 0·28 lb. per cubic inch and the wire is stretched until the sag is reduced to 6 inches. Shew that the tension in the wire is approximately 27220 lbs. per square inch.

2. In launching a ship the motion is checked by the drag of cables which initially descend vertically from the bows and are led away from the ship in the direction of the slipways. If the vertical portion of one such cable is 30 feet and the horizontal portion in contact with the ground is initially 500 feet, find the greatest drag which can be produced by the cable. The coefficient of friction is $\frac{1}{2}$, the weight of the cable is 30 lbs. per foot run.

3. Water is run into the flywheel of an engine to cool a rope brake fitted round its circumference.

The water is carried round in a circle 7 feet in diameter and the flywheel makes 120 revolutions per minute.

Calculate the couple retarding the rotation of the wheel if water is run in at a steady rate of 10 gallons a minute, the velocity of supply being negligible.

4. The crank effort diagram for a double acting single cylinder engine is shewn in Fig. 1. The engine works against a steady resistance. The mass of the flywheel is 3 tons. Its radius of gyration is 3 feet. If the speed of rotation is 120 revs. per minute for the crank position A, calculate the speed at the crank positions B, C and D.

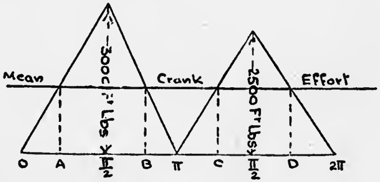


Fig 1

5. The mechanism of a steam-digger is shewn in Fig. 2.

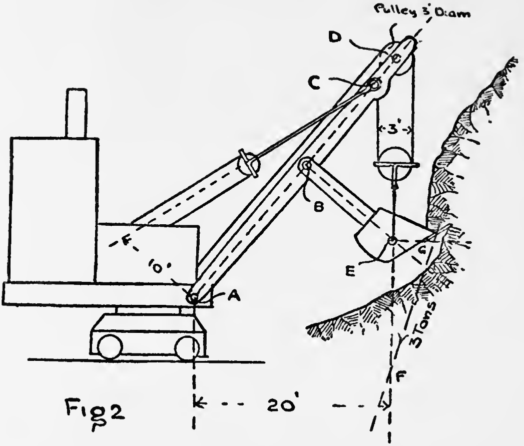


Fig 2

The jib is hinged at A and supported by a tie rod connected to the jib at C. The tension of the winding rope is the same throughout. The force on the cutting edge of the bucket is as shewn on the diagram.

Find the magnitude and direction of the reaction at A and the tension in the supporting tie rod.

Construct the bending moment diagram for the jib.

The weights of the various parts are to be neglected.

Dimensions

$AB = 17'$ $AC = 29'$ $AD = 31'$.

$EG = 8'$ horizontal. $EF = 16'$ vertical.

D is vertically above E.

EBD is a right angle.

The winding rope has one end attached to the point C. It passes round a three-foot pulley and is led away to the winding engine in a direction parallel to AD.

6. In the framework shewn in Fig. 3, the bars AC, BC are freely hinged at C and hinged freely to the points A and B which are to be regarded as absolutely fixed.

Assuming that E for the material of the bars is 30,000,000 pounds per square inch, and that the bars remain perfectly straight, calculate the approximate vertical and horizontal displacements of the point C.

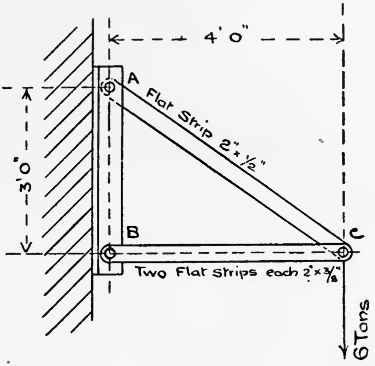


Fig. 3.

7. A spherical shell outside diameter 4 feet and thickness $\frac{1}{2}$ inch is subjected to an internal pressure of 400 lbs. per square inch. Calculate the tensile stress set up in the material and the increase in diameter measured in inches.

$$E = 30 \times 10^6 \text{ lbs. per square inch.} \quad m = \tfrac{10}{3}.$$

8. AB and CD are two steel strips each 2 inches wide and $\frac{1}{2}$ inch deep, CD is 4 feet long and AB is 2′ 6″ long.

AB is laid upon CD and the arrangement is loaded as shewn in Fig. 4. CD is initially straight but AB is initially curved in such a way that when loaded the pressure between

AB and CD is uniformly distributed. Calculate what initial sag must be given to AB to satisfy this condition.

$$E = 30 \times 10^6 \text{ lbs. per square inch.}$$

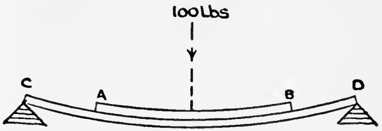


Fig 4

PAPER No. 9.

1. The weight of a locomotive is supported by springs which transmit the pressures to the axles of the wheels as shewn in Fig. 1. The weight supported by the 3 wheels is 15 tons and acts along DE. Taking the dimensions from the figure, find the portion of the weight taken by each wheel.

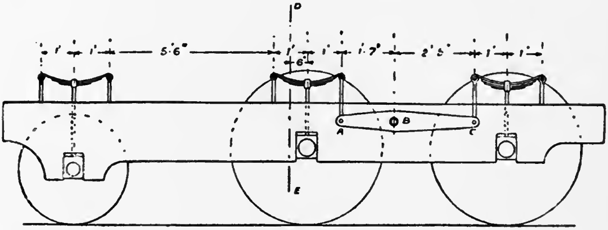


Fig. 1

2. A rocket when full weighs 20 pounds and when all the charge is burnt it weighs 2 pounds. When the rocket is fired in a vertical direction the charge is consumed in 3 seconds and in that time the rocket acquires a velocity of 300 feet per second. Assuming that the charge gives a constant force during the ascent, and that air friction may be neglected, find the value of the force.

3. Two parallel shafts are fitted with spur-wheels having 25 and 61 teeth respectively. The first of these shafts is running at 1000 and the second shaft at 200 revolutions per minute. The moments of inertia of the first and second shafts about their axes of rotation are 50 and 20 in lbs. feet units respectively. If the spur-wheels on the two shafts are made to suddenly engage, calculate the revolutions of the two shafts just after the process and determine the loss in energy due to the shock.

4. In the framework of rods shewn in Fig. 2, *ABCD* and *CEFB* are similar kite-shaped quadrilaterals, each quadrilateral having two pairs of equal sides. The framework is hinged to fixed points *A*, *F* and *B*. Use the principle of work to find the magnitude of the vertical force *P* which will balance *W* acting at *D*.

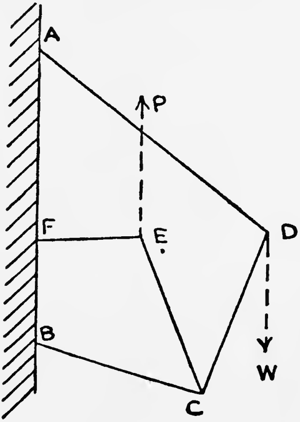


Fig. 2.

5. If R_1, R_2, R_3 are the three principal stresses at a point in an elastic material, shew that the greatest shear stress is $\frac{R_1 - R_3}{2}$, where R_1 is the greatest and R_3 the least of the principal stresses.

6. The cross section of a girder has the following dimensions: Flanges 12″ × 1″, web 24″ × $\frac{1}{2}$″. The girder is bent by concentrated loads and at a section clear of the loads, the shearing force is 100 tons. Calculate what percentage of this shearing force is carried by the web.

If the B.M. at this section is 5000 inch tons, what percentage of this B.M. is carried by the flanges?

7. The two pieces *A* and *B*, shewn in Fig. 3, fit freely into the two ends of a straight tube and are drawn together by a bolt and nut. The section of this tubular distance piece is the same as that of the bolt, and they are constructed out of the same material. The nut is initially screwed up so that the tension in the bolt is 5 tons. The pieces *A* and *B* are then subjected to forces of 3 tons tending to pull them apart. Calculate the resulting tension set up in the bolt.

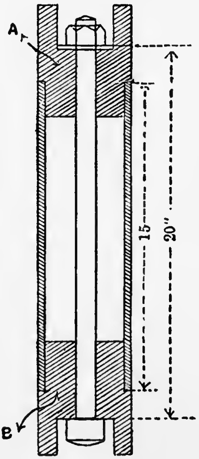


Fig. 3.

8. The horizontal boiler of a motor-waggon is carried between two parallel girders as shewn diagrammatically in Fig. 4. At the smoke-box end A the girders are riveted to the sides of the boiler, and at the other end B the boiler is carried by a pair of brackets riveted to its sides and resting on the top edges of the girders so that the boiler is free to expand. The boiler and girder frame are supported vertically under the points A and C, which are equidistant from B. Assuming that when the girders are unstressed they are just in contact with the brackets at B, prove that the shearing couple on each riveted joint at A is $\frac{1}{8}Wl$ where W is the weight of the boiler, and l the horizontal distance between its centre of gravity and A.

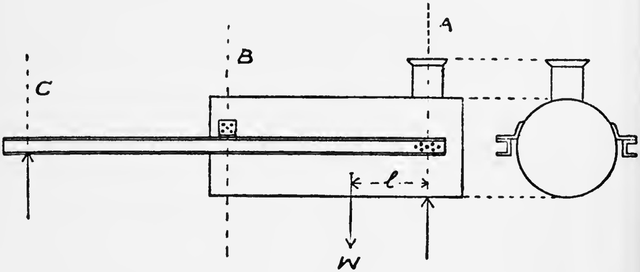


Fig. 4.

PAPER No. 10.

1. Fig. 1 shews the outline of a grip for lifting blocks of stone. The bent link BDC is pivoted at D to the distance piece DE and there are pin joints at A, C, and B. Shew that the horizontal gripping forces are

$$\frac{W}{2}\left[\frac{l}{\sin \alpha} - x\right]\frac{1}{y},$$

where l is the length of the perpendicular from D on AC and x and y are the horizontal and vertical distances of B from D.

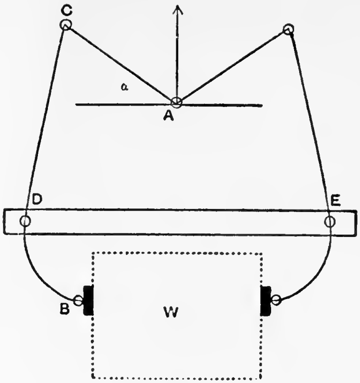


Fig. 1.

2. The cylinder of an hydraulic motor is 6 inches diameter and 12 inches stroke. The pressure in the main is 700 lbs. per square inch and the supply pipe to the motor is 20 feet long and 4 inches in diameter. The piston moves with simple harmonic motion and makes 200 double strokes per minute. Neglect friction of the water and calculate the pressure acting on the piston at the two ends of the stroke.

3. A cubical block of stone, 6 feet on edge, is levered up until its C.G. is vertically above the edge it rests upon. The block is then allowed to fall over, and after striking level ground it begins to rise about an opposite edge. Calculate how high the C.G. of the cube will be carried by this rebound, assuming that the whole blow acts through this opposite edge.

4. A plank AB (see Fig. 2), 20 feet long and weight 50 lbs., rests upon two solid cylindrical rollers P and Q, each one foot in diameter and each weighing 40 lbs. Initially the plank rests with P under A and Q under C, and the system is placed on an

incline of 1 vertical to 10 horizontal. Determine the velocity acquired by the plank when B comes over the roller Q.

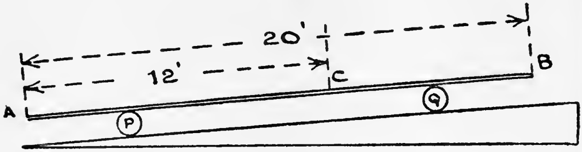


Fig. 2.

5. A bent rod $ABCD$ is fixed horizontally at A in a vertical wall and loaded in the manner shewn in Fig. 3.

The weight of the rod is 3 lbs. per foot run.

Draw to scale the Bending Moment Diagrams for the lengths DC, CB and BA.

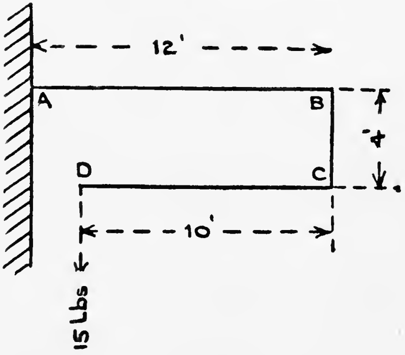


Fig. 3.

6. In Fig. 4 CDE is a beam hinged at its extremity C and supported horizontally by the tie rods AE and BD attached to

the fixed points A and B. Assuming that the beam is absolutely rigid, and that the tie rods are made of the same material, determine the tensile stress set up in each rod.

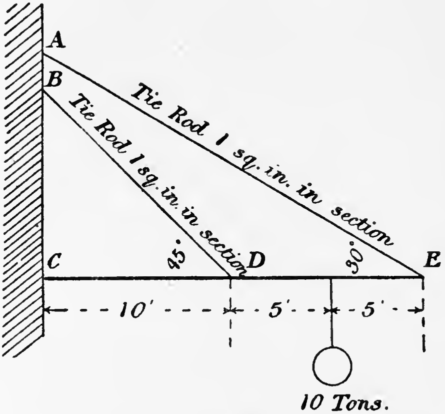


Fig. 4.

7. Two planks each 12″ broad are laid across a 20 foot gap, one plank resting upon the other. Transverse strips are interposed between the planks so that the top plank only bears on the under plank at the middle and end sections. The top plank is 1 inch thick, the under plank is 2 inches thick and a load of 150 lbs. is supported at the middle of the upper plank. Determine the greatest fibre stress set up in each plank.

8. The section of a girder is as follows: Flanges $12'' \times 1''$, web $48'' \times \frac{1}{2}''$. Angles connecting flanges and web $4'' \times 4'' \times \frac{1}{2}''$. The girder is bent by concentrated loads and for a certain length the shearing force is 40 tons. If the total shearing force provided for by each rivet is to be limited to 5 tons, calculate the least permissible pitch of the rivets for this length of the girder.

PAPER No. 11.

1. Fig. 1 represents a band brake operated by a bent lever ACD pivoted at the point C.

$AC = 12''$. $CB = CD = 3''$. The diameter of the brake drum is $12''$.

The coefficient of friction between band and drum is 0·25. A vertical force of 10 lbs. is applied at A, calculate the couple required to rotate the brake drum.

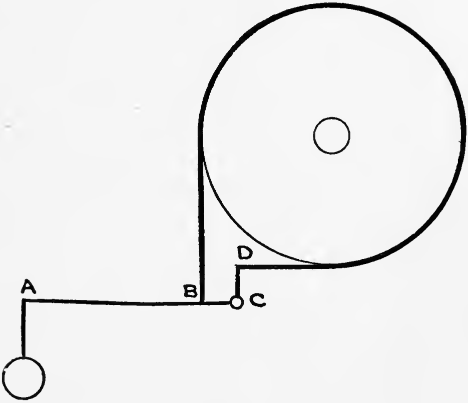


Fig. 1.

2. A straight horizontal tube AB contains a particle mass m attached to the end A by a spiral spring of length l inches. The strength of the spring is such that a pull of m lbs. produces an elongation of d inches. If the tube is caused to rotate with

uniform angular velocity ω about a vertical axis through A, shew that the elongation of the spring is

$$\frac{l}{\frac{12g}{d\omega^2} - 1}.$$

If the particle is set oscillating about the state of steady motion find the period of a complete oscillation.

3. The road resistance experienced by a motor car weighing $37\frac{1}{2}$ cwt. is 60 lbs. and the wind resistance is of the nature av^2 where v is the velocity of the car in feet per second. The maximum speed on the level is 45 miles per hour, the H.P. at the road wheels then amounting to 20. Determine thence the value of the constant a.

The car reaches the bottom of an incline of 1 in 10 at a speed of 45 miles per hour. Find how far it will ascend the slope before its speed falls to 20 miles per hour, the force exerted at the road wheels remaining constant. If the gear is then changed, so that the force at the road wheels is doubled, find the distance run before the speed falls to 10 miles per hour. The gear again is changed, the force at the road wheels being three times the original force, calculate the steady speed at which the car will mount the incline.

4. The current through the armature of a shunt motor is cut off but the field remains fully excited. The armature is brought gradually to rest by friction, hysteresis, eddy currents, and wind resistance. This combined resisting torque is $200 + \frac{1}{4}n + \frac{1}{2000}n^2$ foot poundals, where n is the number of revolutions per minute. If the moment of inertia of the armature about its axis is 1000 lbs. feet^2 and the initial speed of the armature is 600 revolutions per minute, find how long it will take the armature to come to rest.

5. The weight of a cubic foot of earth is w lbs. and the natural angle of repose makes an angle ϕ with the horizontal. Shew that for a layer of this material, extending indefinitely in all horizontal directions, the pressure at a depth h must lie between the limits $wh \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$ and $wh \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$.

6. The truss of a railway bridge consists of 8 panels each 20 feet square. Due to the dead load each lower panel point supports a load of 6 tons as shewn in Fig. 2. If a live load of $\frac{1}{2}$ ton per foot run advances across the bridge, determine how many of the panels need counterbracing.

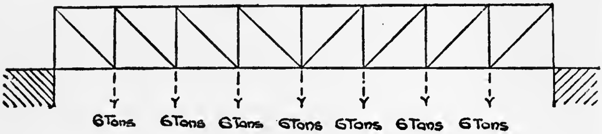


Fig. 2.

7. Fig. 3 is the section of an unequal angle iron. This angle iron is subjected to a bending moment M inch tons, the plane of the bending moment having GY as its trace. Calculate the longitudinal stress at the point P.

The moment of inertia of the section about GU is $3{\cdot}19$ inch^4.

" " " " " " GV is $1{\cdot}54$ inch^4.

$$\tan\theta = {\cdot}55.$$

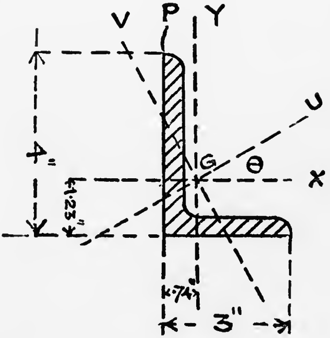


Fig. 3.

8. Fig. 4 illustrates a strut made up of two flat strips $3'' \times 1''$ riveted at their ends to distance pieces holding the strips parallel to one another. Pins pass through these distance pieces in the manner shewn. If these pins are pushed towards one another, the pins remaining parallel and vertically above one another, find what is the greatest load the strut will support without buckling.

$$E = 30 \times 10^6 \text{ lbs. per square inch.}$$

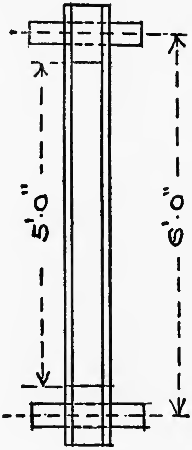


Fig. 4.

PAPER No. 12.

1. In the mechanism shewn in Fig. 1, the part B can slide without friction along AC. If the force P balances W prove that

$$P = \frac{BN \, . \, AB}{BM \, . \, AC} W,$$

where BN and DM are perpendiculars on AD and AC respectively.

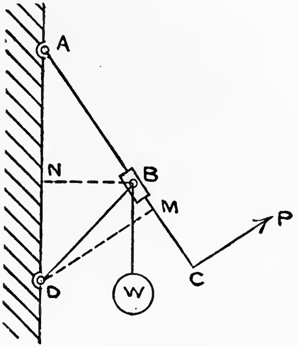


Fig. 1.

2. The brakes are applied with equal force to all the wheels of a four-wheeled railway truck and the back pair are just on the point of skidding. The coefficient of friction between the brakes and the wheels and between the rails and the wheels is 0·18. The height of the C.G. of the truck above rail level is 5 feet, the distance between the axles is 8 feet. Calculate the retardation produced, assuming that the friction between axle and bearing may be neglected. If the wheels are 40 inches in diameter, the journals of the axles 4 inches in diameter, and the coefficient of friction between journal and bearing 0·05, calculate the additional retardation produced thereby.

3. AB, AC, AD, are the legs of a set of shear-legs.

$$AB = AC = 60 \text{ feet}, \quad AD = 90 \text{ feet}.$$

ABC is an isosceles triangle, the base BC is 20 feet and the plane ABC is inclined at 60° to the horizontal. The triangle DBC formed by the feet of the shear-legs is also an isosceles triangle. A pulley wheel is carried at A and a weight of 80 tons is suspended by a rope passing over this pulley and leading to a point half-way between D and BC. Determine graphically or otherwise the stresses in the three legs.

4. A vertical shaft turns in a foot-step bearing. The diameter of the shaft is 12 inches, its end is plane and the coefficient of friction between the end and the bearing is 0·05. If the load on the bearing is 5 tons and the shaft is rotating 120 times per minute, calculate the H.P. absorbed by friction.

5. Fig. 2 illustrates a stand pipe filled with water. The pipe is constructed of $\frac{3}{4}$ inch steel plating. Consider a section 20 feet above the ground level and determine the longitudinal stress across this section and the hoop stress at this level.

The weight of the water is $62\frac{1}{2}$ lbs. per cu. ft.
The weight of the steel is 480 lbs. per cu. ft.

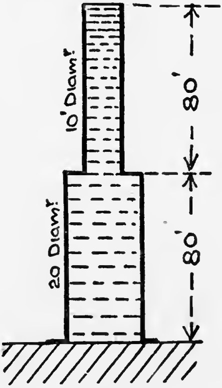
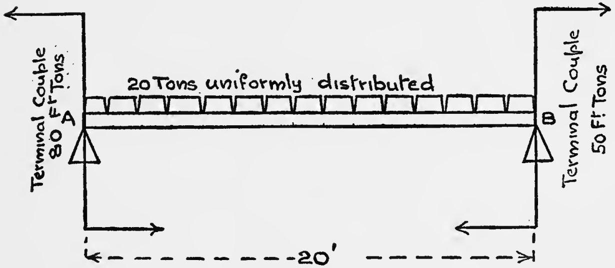


Fig. 2.

6. A beam uniformly loaded rests upon end supports and is subjected to terminal couples as shewn in Fig. 3. Draw to scale the bending moment and shearing force diagrams for the beam.

If this same beam is subjected to a bending moment couple applied at a section distant 12 feet from one end as shewn in Fig. 4, calculate the reactions at the end supports and draw the bending moment and shearing force diagrams for this case.

Fig. 3.



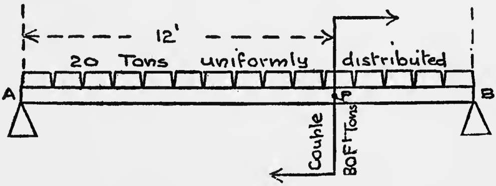


Fig. 4.

7. A hoop 2 inches broad and $\frac{1}{4}$ inch thick is shrunk on to a solid cast iron wheel 24 inches in diameter. The tensile stress in the hoop is 10 tons per square inch: if μ is the coefficient of friction, find the force required to draw off the tyre.

Assuming the cast iron wheel does not compress when the hoop is shrunk on, find how much smaller the tyre should be than the wheel to produce this tensile stress of 10 tons per square inch.

$$E = 14000 \text{ tons per square inch and } m = \tfrac{10}{3}.$$

8. Fig. 5 is an indicator diagram for a double acting steam engine whose stroke is 2 feet and length of connecting rod 4 feet. The area of the piston is 250 square inches. The guide-bars are supported at their extremities, the distance between the supports being 3 feet. Draw the maximum bending moment diagram for the guide-bars for a forward stroke.

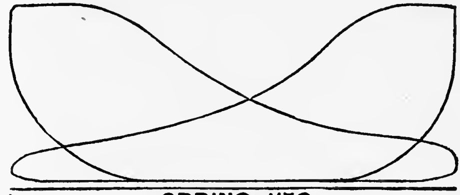


Fig. 5.

PAPER No. 13.

1. Two trucks A and B of mass 6 tons (see Fig. 1) running along parallel level railways are connected by a wire passing round a pulley wheel C. This pulley wheel is held by springs

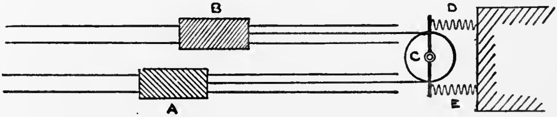


Fig 1

D and E and the strength of these springs is such that each requires a pull of 1 ton to elongate it one inch. The connecting wire being slack, truck A is given a velocity of 4 feet per second to the left. Calculate the velocity of the trucks just after B

has been jerked into motion and the two trucks have acquired the same velocity. Determine the loss of kinetic energy, and, assuming that this is all accounted for by strain energy in the springs D and E, calculate the extension of these springs and the tension in the wire. The energy of rotation of the wheel is assumed to be negligible.

2. A thin plank 8 feet long is laid transversely across two horizontal parallel knife edges 12 inches apart. The centre of the plank is midway between the knife edges. One end of the plank is raised a distance of 3 inches above the level of the knife edges, and then allowed to drop. Find the height to which the other end of the plank will rise.

3. A solid cylinder A of mass 360 lbs., 12 inches in diameter, turns freely in fixed bearings (see Fig. 2). A solid cylinder B of mass 160 lbs., 8 inches in diameter, turns freely in bearings carried on the lever CD hinged at C. When the wheel A is making 800 revs. per minute the lever CD is depressed and the wheel B which is at rest is brought into contact with A. Determine the revolutions of the wheels when the speed has again become constant. If the pressure between the wheels is 200 lbs. and the coefficient of friction is 0·25, determine how long it takes to effect the equalizing process.

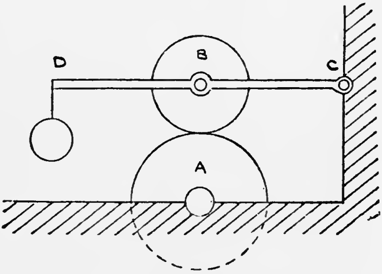


Fig. 2.

4. A water wheel has a series of buckets mounted on its rim, the entering lip of the bucket is tangential to the rim of

the wheel and the discharging lip makes an angle θ with the rim. The rim velocity is v and water impinges on the bucket in the direction of the tangent with velocity u. Find the change of momentum per pound of water in the direction of the tangent and shew that the efficiency of the wheel is

$$\frac{2v(u-v)(1+\cos\theta)}{u^2}.$$

Deduce the ratio of u to v for maximum efficiency in the case of a Pelton wheel for which the angle θ is zero.

5. At the outside layers of a propeller shaft, the shear stress is 4 tons per square inch and the longitudinal compressive stress is 6 tons per square inch.

A sample of the material in its natural elastic state is subjected to a simple tensile test and its limit of linear elasticity occurs when the tensile stress is 16 tons per square inch.

Determine the factor of safety corresponding to the three recognised theories of elastic breakdown.

Poisson's ratio is to be taken as $\frac{3}{10}$.

6. Draw the bending moment and shearing force diagrams for the beam shewn in Fig. 3. The beam is supported horizontally by the strut DE, hinged at one end to a wall and at the other end to the projection CD which is firmly fixed at right angles to AB. The beam AB is freely hinged to the wall at B. The weight of the beam and strut can be neglected.

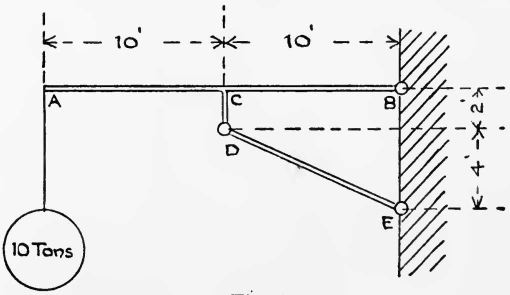


Fig. 3.

7. A vertical masonry chimney has an internal diameter d_1 and an external diameter d_0. The base of the chimney is given a horizontal acceleration of α feet per sec. per sec. and the whole chimney moves horizontally with this acceleration. Shew that at a section of depth h below the top of the chimney the resultant stress acts at a distance $\dfrac{\alpha h}{2g}$ from the centre of the section. If the chimney behaves as an elastic solid, shew that at a depth $\dfrac{g\,(d_0^2 + d_1^2)}{4\alpha d_0}$ below the top, horizontal cross sections will be subjected to normal tensile stress.

8. In the bridge shewn in Fig. 4 the horizontal member is made up of separate bars *ED, DC, CB*, etc., hinged together at their extremities. Use the method of sections to determine the stresses in the members *AB, BC*. The tension stresses in the bars *ED* and *GH* may be taken as 2 tons. Draw the reciprocal figure for the whole framework and determine the horizontal and vertical components of the reaction at *A*.

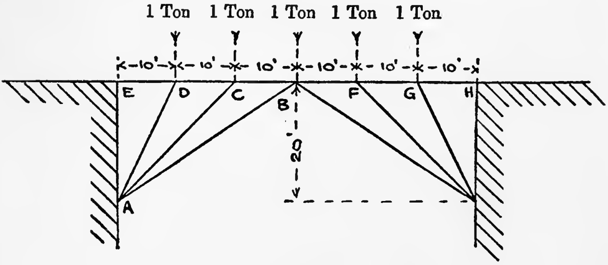


Fig. 4.

PAPER No. 14.

1. A and B are two horizontal cylinders (see Fig. 1) turning freely in fixed bearings, A making one revolution per second and B one revolution in two seconds. The mass of each cylinder is 40 lbs., the radius is 6 inches and the radius of gyration is 4 inches. A plank C weighing 80 lbs. is then gently laid across the cylinders. Calculate the velocity given to the plank when the velocities have once again become steady. Taking the coefficient of friction between plank and cylinder as $\frac{1}{2}$, determine how long this equalizing process takes, assuming that the weight supported on each cylinder is 40 lbs.

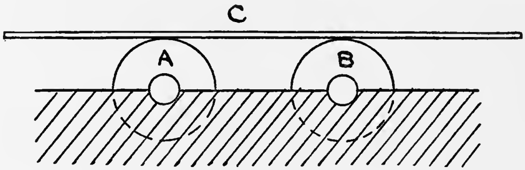


Fig. 1.

2. A pontoon of rectangular form, 12 feet long and 4 feet wide, weighs 1000 lbs. and is floating in fresh water. Find the position of the transverse metacentre and determine the righting moment for a transverse heel of 5°, if the centre of gravity be 1 foot above the bottom.

If a man weighing 150 lbs. stands on the centre line of this pontoon at a distance of 2 feet from one end, calculate the draught of the pontoon at each end.

3. Fig. 2 illustrates a friction drive. The shaft A drives the shaft B by means of a cylindrical wheel C pressing upon the disc wheel D. If there is no slipping between the wheels at the radius r_0 and the pressure between them is uniformly distributed along the line of contact, shew that the efficiency of transmission, neglecting losses at the bearings, is $\dfrac{r_0 + r_1}{2r_0}$.

If the radius of "no slip" is r, shew that

$$r = \frac{r_0 + r_1}{2} + \frac{T(r_0 - r_1)}{2\mu Pa},$$

and that the efficiency of transmission is

$$\frac{2r^2 - (r_1^2 + r_0^2)}{4r^2 - 2r(r_1 + r_0)},$$

where T is the couple applied to shaft A,

a is the radius of the wheel on shaft A,

P is the total pressure between the wheels,

μ is the coefficient of friction.

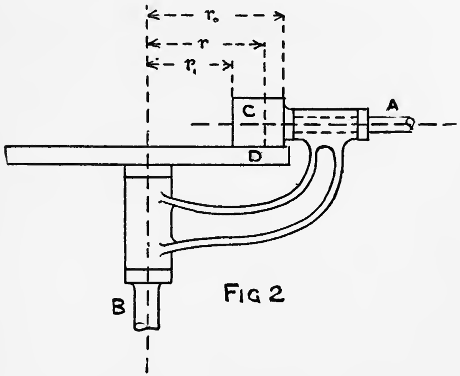


FIG 2

4. The exhaust valve of a petrol motor weighs 3 oz. and the lift of the valve is $\frac{1}{4}''$: the engine speed is 1500 revs. per min. The cam is so shaped that the valve would be closed in $\frac{1}{10}$th of a revolution, descending with uniform velocity, if the valve stem remained in contact with the cam, but, owing to inertia, the valve is left behind. Assuming a uniform valve spring pressure of 30 lbs. wt. tending to close the valve, find the distance through which the valve falls before it overtakes the cam, and the relative velocity at that instant.

5. A steel shaft, 1 inch in diameter, is provided with enlarged portions P and Q, $1\frac{1}{2}$ inches in diameter. The shaft is held twisted by an axial torque of 40 feet-lbs. While in this condition a steel tube, $\frac{1}{20}$-inch thick, is shrunk on to the enlarged portion as shewn in Fig. 3. When the tube has firmly gripped the shaft the applied torque is removed. Calculate what twisting couple remains in the shaft, assuming that the shaft and tube are made of the same material.

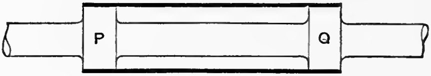


Fig. 3.

6. A ring is supported by 4 wires AB, CD, EF, GH (see Fig. 4). Each wire is 40 inches long, 0·1 square inch in section and is initially stretched to a tension of 1 ton. A load of 3000 lbs. is then hung from the ring, find the tension produced in each wire and the vertical deflection of the load.

Assume that the points A, B, C, D, are absolutely fixed and that E for the wire is 30×10^6 lbs. per square inch.

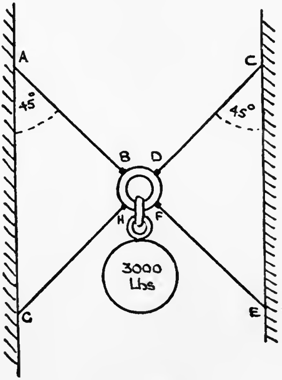


Fig. 4.

7. A shaft one inch in diameter and 3 feet long has its ends rigidly fixed. An axial torque of 2000 inch-lbs. is applied to the shaft at a section distant 12 inches from one end. Determine the magnitudes of the axial couples set up at the two ends, and the greatest shear stress in the material.

8. A timber beam loaded at the centre and supported at the ends deflects $d = \dfrac{W_0 l^3}{48 E_2 I}$ where E_2 is the modulus of elasticity for the wood. The beam is trussed by two steel tie rods each of section A and modulus of elasticity E_1, and a single central pillar of height $\frac{1}{2}nl$. Shew that the load W required to produce the same deflection d is now increased to

$$W = W_0 \left[1 + \frac{E_1 A}{12 E_2 I} \frac{n^2 l^2}{(n^2 + 1)^{\frac{3}{2}}} \right].$$

The yielding of the central strut may be neglected.

PAPER No. 15.

1. Fig. 1 represents a table supported upon two trestles hinged to the table at C and D. Use the principle of work to determine the magnitude of the force P which will just cause the trestles to turn about the points A and B.

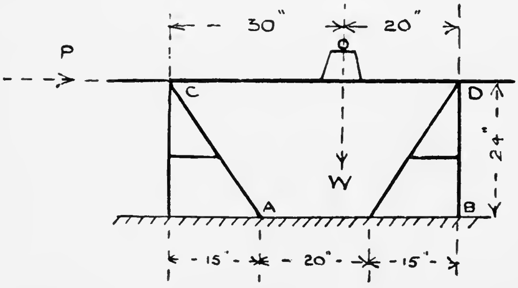


Fig. 1.

2. A mass resting upon the ground is attached to the lower end of an unstretched vertical spiral spring. The upper end of the spring is then given a constant upward velocity of v feet per second. After this upper end has been raised a distance of d feet the mass commences to rise. If x is the distance moved by the mass in a further time t, shew that the motion is given by the equation

$$\frac{d^2x}{dt^2}+\frac{g}{d}x=\frac{g}{d}vt.$$

Hence prove that the greatest extension of the spring is

$$d+v\sqrt{\frac{d}{g}}.$$

3. The wheel shewn in Fig. 2 is out of balance due to a hole bored in the rim which has removed $\frac{1}{2}$ lb. of metal. The total weight of the wheel and the framework on which it rests is 200 lbs. Calculate how many revolutions per minute the wheel can make without causing the frame to rock on its supports.

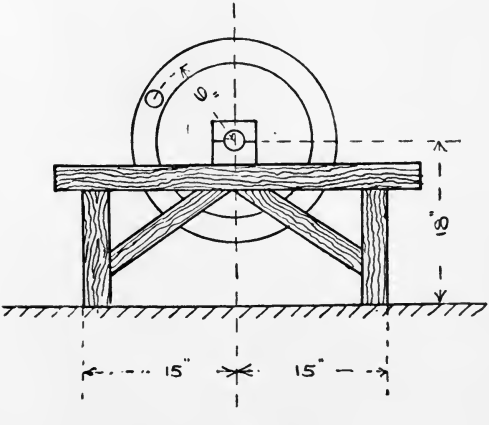


Fig. 2.

4. A lifting tackle consists of upper and lower multiple pulley blocks. The upper block contains five pulleys, the lower block four, and nine ropes connect the two blocks. What is the mechanical advantage of the arrangement, neglecting friction?

It is found that to lift 100 lbs., a force of 20 lbs. must be applied, and to lift 200 lbs., the force required is 32 lbs. Deduce the force required to lift 300 lbs., and state the efficiency of the tackle for each of these three cases.

5. A split piston ring for a cylinder 4 inches in diameter, is $\frac{1}{4}$ of an inch deep. When sprung into the cylinder it exerts a uniformly distributed pressure of 10 lbs. per square inch. Draw the bending moment diagram for the ring developed out into a straight line.

If the section of the ring opposite the split is $\frac{3}{16}$ inch thick determine the maximum tension in the material at this section.

6. A wooden beam 24 feet long is supported at its extremities and carries a central load of 40 tons. The beam consists of two 12″ × 12″ timbers laid one above the other and bolted together with vertical tie bolts.

The longitudinal shearing action between the timbers is provided for, by cross pieces 4″ × 4″ × 12″ spaced 24″ apart as shewn in Fig. 3. Assuming that the cross pieces take all the shearing action between the timbers deduce the intensity of the shearing stress in each cross piece.

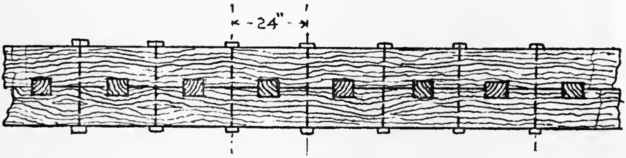


Fig. 3

7. The cylindrical pieces A and B (see Fig. 4) are joined by two steel rods having their ends firmly fixed in A and B.

Calculate the couple required to twist B relatively to A about the axis PQ through a small angle θ radians. C for the rods is $12{\cdot}5 \times 10^6$ lbs. per square inch, E is 30×10^6 lbs. per square inch.

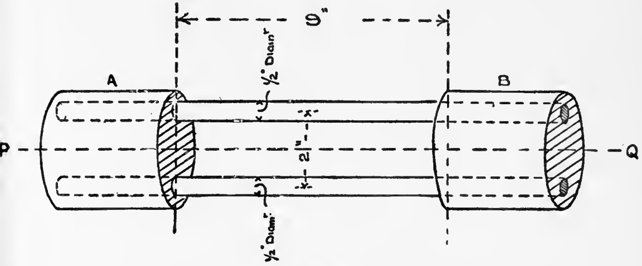


Fig. 4.

8. A flywheel whose moment of inertia is 100 ft. lbs. is mounted on a horizontal shaft 2 inches in diameter. Initially the wheel and shaft is at rest. One end which is 40 inches from the wheel is then started rotating at a steady speed. If C for the material is $12{\cdot}5 \times 10^6$ lbs. per square inch, determine the least value of this steady angular velocity which will overstrain the shaft, assuming that elastic breakdown occurs for a shearing stress of 20,000 lbs. per square inch.

PAPER No. 16.

1. Fig. 1 illustrates a form of steering gear, which in a particular case has the following dimensions: length of cross-head 3 feet, length of each connecting rod 3 feet 6 inches, distance of the joints on the nuts from the centre line 12 inches, single threaded right and left-handed screw with four threads to the inch.

Neglect friction and determine the ratio between the couple exerted on the rudder and the couple applied to the screw

(*a*) when the cross-head has the position shewn;

(*b*) when the cross-head has turned through 30°.

(*c*) Find also how much the screw has shifted longitudinally during this rotation of the cross-head.

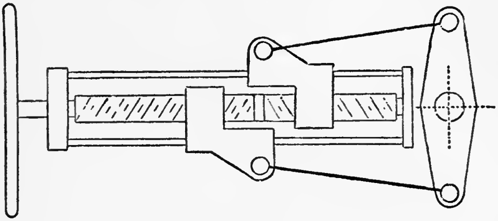


Fig. 1.

2. A pair of wheels and axle of a railway truck carry a load exerted through the springs amounting to 3 tons. The mass of the wheels and axle is 800 lbs. The truck is running at a speed of 45 miles an hour along a track which is worn into slight undulations, sinusoidal in form. The length of the undulations measured from crest to crest is 6 feet and the height measured from crest to hollow is 0·1 inch.

Assuming that the load exerted by the springs remains unchanged, determine the greatest and least pressures between the rails and wheels.

3. In the governor shewn in outline in figure 2 the sleeve BD is free to slide up and down on the spindle KL and rotates with it. The bell-crank lever CBA carries the ball at A and a roller at C. The control is exercised by the weights of two balls, each W_1 lbs., the weight of the sleeve, W_2 lbs., and the thrust S lbs., of a spring in compression, placed in the annular

space between the spindle and the sleeve. Shew that in any position the radial force at the ball centre necessary for equilibrium is

$$\left(\frac{S+W_2}{2}\right)\frac{IB}{AN}+W_1\frac{IN}{AN}.$$

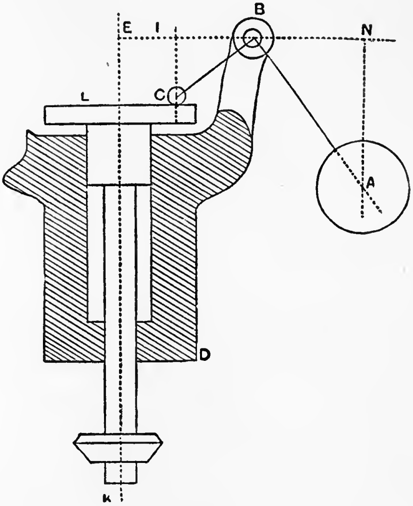


Fig. 2.

4. The flywheel of a motor-car engine has a mass of 120 lbs. and its radius of gyration about the axis of rotation is 9 inches. This axis of rotation is horizontal and set across the car. With the engine making 1500 revs. per minute the car takes a corner of 60 feet radius at 30 miles an hour. Calculate the magnitude of the gyroscopic couple due to the flywheel and if the top of the flywheel is moving backwards relatively to the car, shew that the gyroscopic action tends to steady the car on the turn, no matter whether the turn is made to the left or right.

5. Fig. 3 illustrates a spring attached to a frame by means of an inclined link BC. AB is 50 inches and the point D is 8 inches below AB. Determine the bending moment at the mid-section of the spring when the link BC makes an angle θ with the vertical, and examine the value of θ which makes this bending moment a minimum.

Shew that the bending moment at any section of the spring is proportional to the vertical intercept between the centre line of the spring and the lines BE and EA.

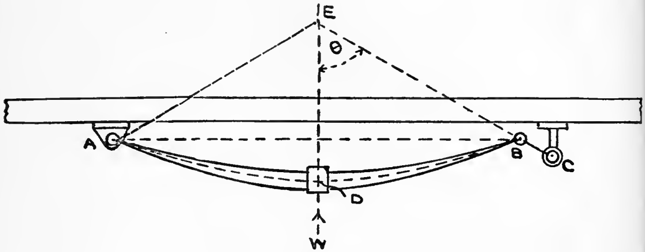


Fig. 3.

6. The rim of a wheel is a strip 2 inches wide and $\frac{1}{8}$ inch thick. The wheel is 3 feet in diameter and has a hub 3 inches in diameter. The rim and hub are connected by 24 spokes, each 0·1 square inch in section, screwed up to an initial tension of 1000 lbs. The rim and the spokes are made of the same steel which weighs $\frac{1}{4}$ lb. per cubic inch.

If the wheel is set rotating 1000 revs. per minute determine the stresses set up in the spokes.

7. In excavating a shaft the earth is held back by vertical boards kept in position by timber frames spaced at frequent intervals down the shaft. One of these frames is shewn in the accompanying sketch. The four timbers AB, BC, CD, DA are subjected to uniformly distributed loads amounting to 4 tons

per foot arising from the earth thrust. These timbers are strengthened by the arrangement of struts HE, EF, FG, GH. Assuming that the struts do not alter in length and that the four timbers AB, BC, CD, DA are simply supported at their extremities, their unsupported lengths being 40, 30, 40, 30 feet respectively, calculate the thrusts which must be taken by each of the four struts.

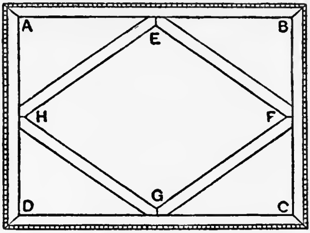


Fig. 4.

8. A circular disc of mass 16 lbs., is fitted with a spindle $\frac{1}{2}$ inch in diameter. This spindle rests with its ends in bearings 12 inches apart and the disc is midway between them. If the C.G. of the disc is $\frac{1}{100}$th inch out of centre and the bearings are not sufficiently tight to encaster the ends, calculate the deflection produced in the spindle for any given angular velocity ω, taking E to be 30×10^6 lbs. per square inch.

If the longitudinal stress in the shaft is to be limited to 5 tons per square inch, shew that the speed may lie below a certain value or above another value but may not have a value lying between. Calculate these two limiting speeds for this particular case.

PAPER No. 17.

1. A rectangular sluice-gate, 4 feet high and 3 feet wide, can slide up and down between vertical guides. It is operated by a screw by means of which a thrust of 400 lbs. can be applied vertically in the centre line of the sluice. The gate when nearly closed is brought up against an obstruction at a point 6 inches from one end of the lower edge. If the coefficient of friction between the vertical edges and the guides is ·25, what thrust is applied to crush the obstruction?

2. Water is found to have gathered behind the reservoir wall as shewn in fig. 1. Determine the point in AB at which

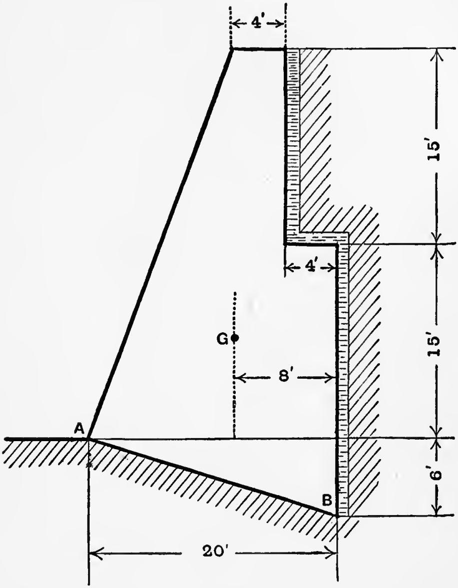


Fig. 1.

the resultant pressure of the wall upon its base acts. The weight of the wall acts in the vertical through G and is 20 tons per foot run of length. The reservoir is empty.

3. The rim of a flywheel weighs 2000 lbs. and its diameter is 5 feet. The plane of the wheel when keyed to the shaft is out of truth by 1°. If the axis of the shaft passes through the C.G. of the wheel, shew that the torque on the bearings when the wheel is rotating at 180 revs. per minute is about 1230 lbs. feet.

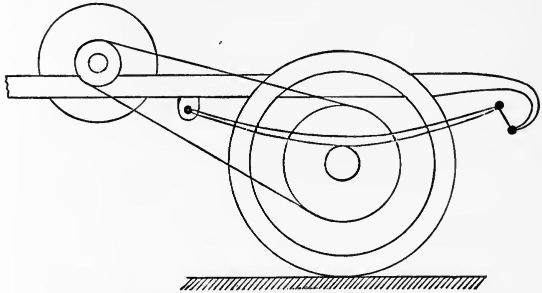


Fig. 2.

4. The crank-shaft of a motor-car engine with a heavy flywheel is set transversely to the frame and drives the back axle by a direct chain drive as shewn in fig. 2. The front sprocket wheel is connected to the engine shaft by means of a friction clutch. The car is standing still with the engine running light at angular velocity ω, when the clutch is suddenly let in, shew that the momentum of the flywheel will impart to the car a velocity

$$\frac{\rho R I \omega}{M R^2 + \rho^2 I},$$

where I is the moment of inertia of the flywheel about its axis of rotation,

M is the total mass of the car,

R is the radius of the road wheels,

ρ is the ratio of the diameters of the rear and front sprockets.

5. The framework illustrated in fig. 3 is supported by vertical reactions at A and D. Use the method of section to determine the stress in BC

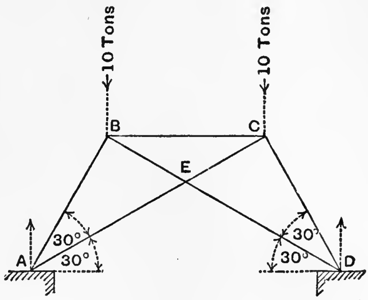


Fig. 3.

If the sections of the beams are such that struts and ties are both stressed to $3\frac{1}{2}$ tons per square inch and E is 14,000 tons per square inch, prove that the increase in the span AD due to the load is $\frac{1}{400}$ BC.

6. A rectangular concrete block rests on a horizontal bed. Its upper surface carries a load of 5 tons per square foot and its sides are subjected to a thrust of 8 tons per square foot. Deduce the shear stress and normal stress for a plane passing through one of the top edges, inclined at an angle of 30° to the horizontal.

If E for the material is 2×10^6 lbs. per square inch and Poisson's ratio is $\frac{1}{3}$ determine the stretch in a direction normal to the plane mentioned above.

7. Fig. 4 represents a concrete beam reinforced with steel rods. When fully loaded the compressive stress in the concrete is not to exceed 600 lbs. per square inch and the tensile stress

in the steel is not to exceed 16,000 lbs. per square inch. If Kd is the depth of the neutral axis, shew that, in order to fulfil the above conditions, K should have the value 0·36.

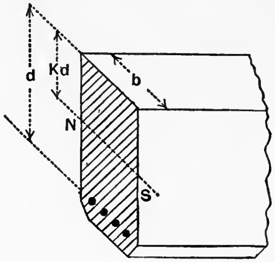


Fig. 4.

E for the steel and concrete is 30×10^6 and 2×10^6 lbs. per square inch respectively, the tensile stress of the concrete is to be neglected and the longitudinal stretch is supposed to vary uniformly over the cross section.

If $b = 8''$ and $d = 12''$ calculate the sectional area of the reinforcing steel rods and the bending moment the beam can withstand without exceeding the working stresses mentioned above.

8. A hollow propeller shaft has to be designed to meet the following requirements:—Its weight is not to exceed W lbs. per inch run; the shear stress under a torque of T inch-lbs. is not to exceed S lbs. per square inch. If the weight of the material is $\frac{1}{4}$ lb. per cubic inch, shew that the value of d_0 the outside diameter of the shaft is given by the quadratic

$$d_0^2 - \frac{Td_0}{2WS} = \frac{8W}{\pi},$$

PAPER No. 18.

1. Fig. 1 illustrates the mechanism of the oscillating cylinder engine. If the crank A rotates with uniform angular velocity ω determine the angular velocity of the cylinder in the position shewn.

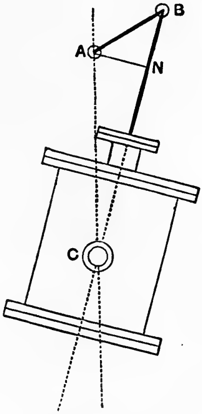


Fig. **1.**

Prove that the acceleration of the piston relatively to the cylinder is $\frac{BN.NC}{BC}\omega^2$, where AN is drawn perpendicular to BC.

2. Fig 2 illustrates a pair of wheels and axle standing on longitudinal rails shaped to accurately fit the conical treads

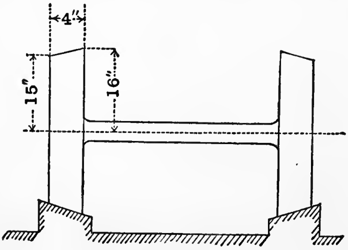


Fig. 2.

of the wheels. If the weight of the wheels and axle is 300 lbs. and the coefficient of friction between the wheels and rail is 0·15 calculate the least force applied horizontally through the centre of the axle which will set the system in motion.

3. A spiral spring hung up by one end has a load of 10 lbs. attached to the other whereby it is given an elongation of $\frac{3}{8}$ inch. The upper end of the spring is then caused to move up and down 240 times a minute with a simple harmonic motion having a travel of $\frac{1}{2}$ inch. Determine the travel of the motion of the suspended mass when this motion has become steady.

4. A four-cylinder petrol engine has a flywheel on one end of the crank-shaft and a clutch on the other, the moments of inertia of flywheel and clutch being respectively 60 and 15 ft.^{2}-lbs. and the distance between them 2 feet 6 inches. The stiffness of the crank-shaft is such that a torque of 1000 inch-lbs. is required to twist one foot length through one degree. Calculate the speed of rotation at which the frequency of the engine impulses will synchronize with the frequency of the torsional oscillations of the shaft, neglecting the mass of the shaft and cranks.

5. One pulley wheel drives another by means of an endless steel band one inch wide and 0·025 inch thick. The tensions of the taut and slack sides are 250 and 50 lbs. respectively and the speed of the taut side is 60 feet per sec. If the driving and driven wheels have the speeds of the taut and slack sides of the band respectively determine the energy lost per sec. in transmission and find how far this can be accounted for by a dissipation of the strain-energy put into the material as it passes from the slack to the taut side. E for the steel band is 30×10^6 lbs. per square inch.

6. A steel band passes round a pulley mounted on a fixed horizontal axis, the ends of the band being rigidly anchored as shewn in fig. 3. The band is initially stretched to a tension of 200 lbs. If the coefficient of friction between the band and the wheel is 0·25 calculate what couple would be required to rotate the wheel in a clockwise direction.

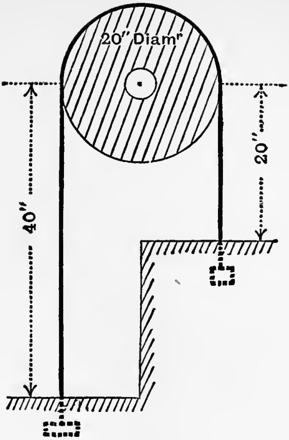


Fig. 3.

7. A circular steel ring of mean diameter 6 inches has a depth of $\frac{1}{2}$ inch and a radial thickness of $\frac{1}{8}$ inch. The ring is cut through at one section and the ends are forced apart a distance of $\frac{1}{25}$ inch by forces acting on the ends in the direction of the tangent to the ring. If E for the material is 30×10^6 lbs. per square inch determine the magnitude of these forces and the greatest stress set up in the material.

8. Figs. 4 a, 4 b represent a rod encastered in a forked knife edge. The free length of the rod is l, and the length of each leg of the forked knife edge is d. If the rod is subjected to an axial pull P, prove that instability will occur and a deformation of the type indicated in fig. 4 c will take place when P reaches the value given by the equation

$$\tanh al = da$$

where

$$a = \sqrt{\frac{P}{EI}}.$$

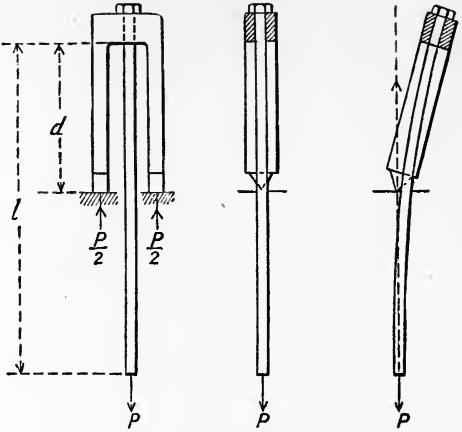


Fig. 4 *a*. Fig. 4 *b*. Fig. 4 *c*.

PAPER No. 19.

1. A brake mechanism is shewn in fig. 1, the two shoes *A* and *B* being rigidly attached to the levers *K* and *L* which are pivoted at the fixed points *E* and *F*. Assuming that the

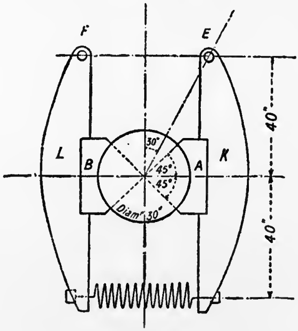


Fig. 1.

normal wear on the shoes is proportional to the power consumed in friction per unit area of bearing surface, shew that after the shoes have been in use for some time the distribution of pressure along the circumference of the wheel is sinusoidal. Locate the position of maximum normal pressure, and calculate the braking couple if the coefficient of friction is 0·2 and the pull of the spring is 100 lbs.

2. The loaded piston shewn in fig. 2 is held in position in a vertical cylinder by a spring whose strength is such that a 100 lbs. pull compresses it a distance of 1 inch. The weight of the piston and load is 150 lbs. and the friction of the piston in the cylinder is such that it takes a pull of 40 lbs. to move the piston. If the piston is depressed until the compression of the spring is 3 inches and is then released, calculate how far the piston will rise and where it will eventually come to rest.

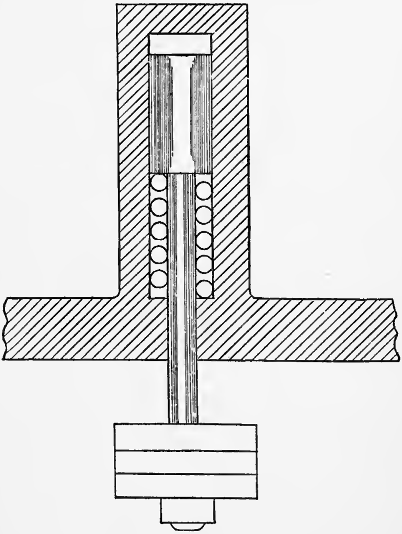


Fig. 2.

3. The door of a railway carriage is 2 feet 6 inches wide. The door is standing open at right angles to the carriage when the train starts with an acceleration of 0·5 feet per sec. per sec. Calculate the angular velocity of the door at the instant it closes.

4. The weight of 20 lbs. shewn in fig. 3 is supported by the post A and the triangular framework which carries freely turning rollers at B and C. The mass of the frame and rollers is 5 lbs., the moment of inertia of the roller B about its axis of rotation is 2 lbs. inch^2; the small and large radii of B are $1\frac{1}{2}$ inches and 4 inches respectively, and the mass of the roller C is negligible. Determine the acceleration with which the frame climbs the post assuming that the roller B does not slip.

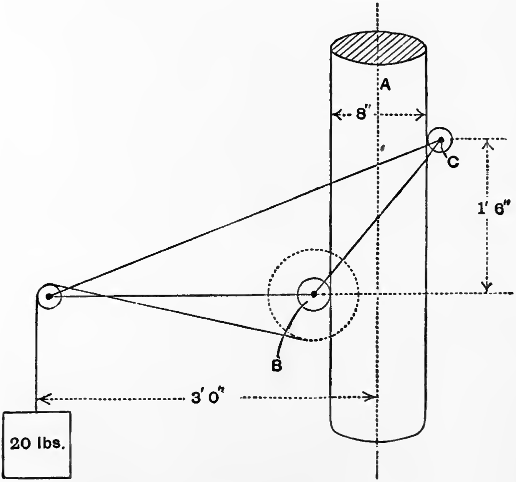


Fig. 3.

5. The load distribution (full lines) and upward water thrust (dotted lines) for a ship are given in fig. 4, the numbers indicating tons per foot run. By the method of the funicular polygon or otherwise, draw the bending moment and shearing force diagrams for the ship.

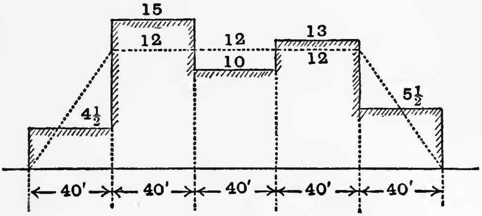


Fig. 4.

6. The weights upon the two axles of a traction-engine are W_1 and W_2. Shew that as the engine passes over a bridge the maximum bending moment at any particular section occurs when one or other of the axles is directly over the section. Determine for what portion of the bridge the maximum bending moment occurs under the load W_1.

If W_1 is greater than W_2 shew that the greatest bending moment at any section in the length of the bridge is

$$\frac{[W_1 l + W_2(l-a)]^2}{4l(W_1+W_2)},$$

where l is the span and a the distance between the axles.

7. Two timber beams $6'' \times 6''$ are encastered horizontally in the manner shewn in fig. 5 *a*. Take E for the wood as 2×10^6 lbs. per square inch and calculate

(*a*) The pressure on the central support.

(*b*) The relative longitudinal movement of the upper beam and the central support.

(*c*) If this relative movement is prevented by cross pieces as shewn in fig. 5 *b* determine the shear stress measured in lbs. per square inch which each cross piece has to withstand. The horizontal section of each cross piece is $6'' \times 6''$.

Fig. 5 *a*.

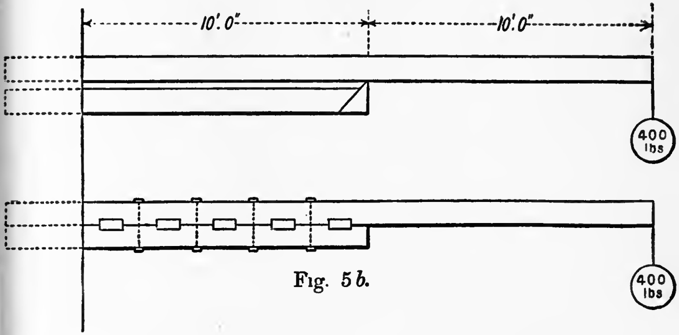


Fig. 5 *b*.

8. A square framework is formed of four straight rods of length l joined rigidly together at their extremities. Two opposite corners are pulled apart by forces of magnitude P. Shew that the elongation of the corresponding diagonal is $\frac{Pl^3}{12EI}$, where I is the moment of inertia of the cross section of each rod.

PAPER No. 20

1. A simple form of "hit and miss" governor for a gas engine is illustrated in fig. 1. The rod A is moved to and fro by means of a cam and carries with it the bent lever C having a solid ball B at one extremity. If the forward acceleration of A is sufficiently great, the inertia of the ball causes the end of C to fall clear of the push piece D which operates the gas inlet valve.

A miss occurs if the end of C falls $\frac{1}{8}$ inch while A moves forward $\frac{1}{4}$ inch from its starting position. If the acceleration of A during this motion is constant and of magnitude α, calculate the least value of α which will cause a miss to occur. The mass of the bent lever may be neglected compared with the mass of the ball.

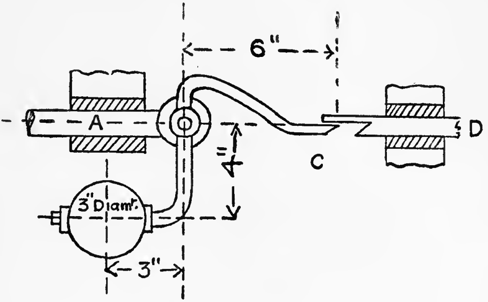


Fig 1

2. A and B are two spiral springs hung and loaded in the manner shewn in fig. 2. The strengths of the springs are

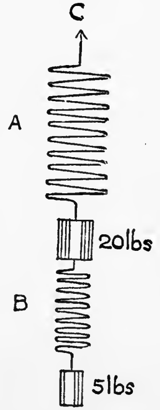


Fig (2)

such that a pull of 10 lbs. extends A and B $\frac{3}{4}$ inch and 3 inches respectively. If the point C is given a vertical simple harmonic motion, write down the equation for the motion of the 20 lb. mass and shew that if the impressed motion has a particular periodicity the 20 lb. mass will come to rest when the state of steady motion is attained.

3. Fig. 3 represents a hand-cart which is wheeled along a cement pavement having the corrugated form shewn in the section. The total mass of the truck is 300 lbs., the mass of each wheel is 30 lbs., the radius of gyration and diameter of each wheel are $1\frac{1}{2}$ and $3\frac{1}{2}$ feet respectively. The truck is wheeled with its platform horizontal at an average speed of 3 feet per sec. Calculate the expenditure of energy necessary to maintain this motion due to the succession of impacts.

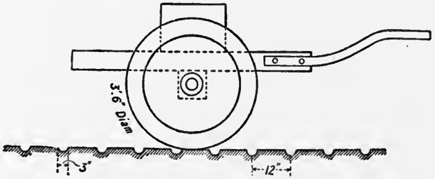


Fig. 3.

4. For the outside cylinder locomotive diagrammatically represented in fig. 4 a close approximation to the inertia effects of the connecting rod is obtained by considering its mass to consist of 80 lbs. and 240 lbs. concentrated at A and B respectively. The length of the C.R. is 6 feet, the length of the crank is 12 inches, the diameter of the driving wheels is 5 feet, the combined mass of the piston, piston-rod and cross-head is 170 lbs., the downward pressure on the wheel exerted by the spring is 20,000 lbs., and the engine is running at a speed of 60 miles per hour. Consider the position defined by $ACB = 45°$, in which the thrust on the piston due to steam pressure is 10,000 lbs.

Calculate (*a*) the value of H the horizontal thrust at the main bearing.

(*b*) The propulsive force on the locomotive due to this one cylinder.

(*c*) The value of V the normal pressure between rail and wheel.

(*d*) The value of S the tangential force between rail and wheel.

The mass of the crank-pin and the mass of 240 lbs. concentrated at B are supposed to be balanced by a suitable counterweight.

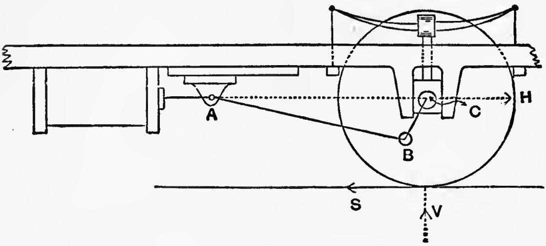


Fig. 4.

5. Describe a general method for determining stresses in a pin-jointed framework which contains one or more redundant bars and is loaded at its joints in a given manner.

Consider the double cantilever framework illustrated in fig. 5 and determine the stress given to the member AB by the vertical loads applied at E and F.

The numbers against the bars give their sectional area in square inches.

The reactions at C and D are vertical.

E is the same for all the bars.

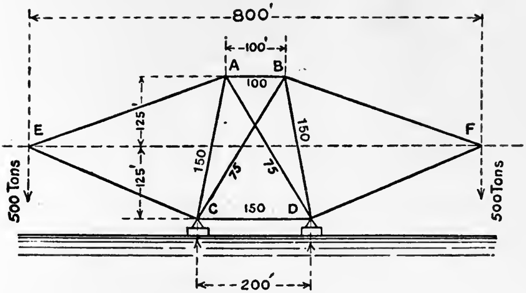


Fig. 5.

6. The two-pin parabolic arch *ADG* represented in fig. 6 has a span of 240 feet and a rise of 48 feet. Due to the roadway vertical forces of 20 tons act on the arch at the points *B, C, D, E, F*. Assuming that the arch is of uniform section throughout and that the abutments are perfectly rigid, determine the horizontal thrust at the hinges, and draw the bending moment diagram for the arch.

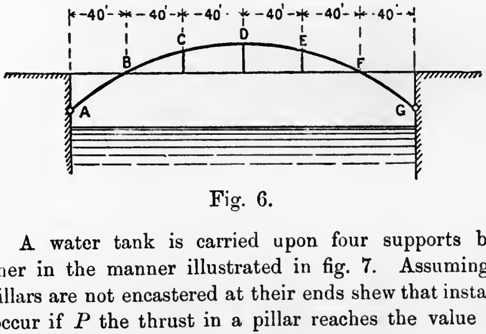


Fig. 6.

7. A water tank is carried upon four supports braced together in the manner illustrated in fig. 7. Assuming that the pillars are not encastered at their ends shew that instability will occur if P the thrust in a pillar reaches the value given by the equation

$$\frac{\sin a\alpha \sin b\alpha}{\sin l\alpha} = \frac{ab}{l}\alpha,$$

where
$$\alpha = \sqrt{\frac{P}{EI}}.$$

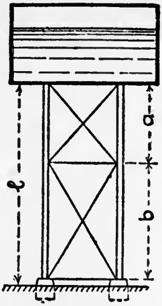


Fig. 7.

8. Fig. 8 represents the cross-head of a steam engine. Owing to insufficient lubrication of the slide and pin, the reaction of the slide is displaced into the position shewn. The thrust in the piston-rod being axial and of magnitude 8 tons, determine the position and magnitude of the thrust on the section AB of the connecting rod taken 18 inches from the centre of the pin.

If the section of the C.R. is a circle 4 inches in diameter, determine the greatest compressive stress set up. The big-end of the C.R. is supposed to be free from friction and the weight of the C.R. is to be neglected.

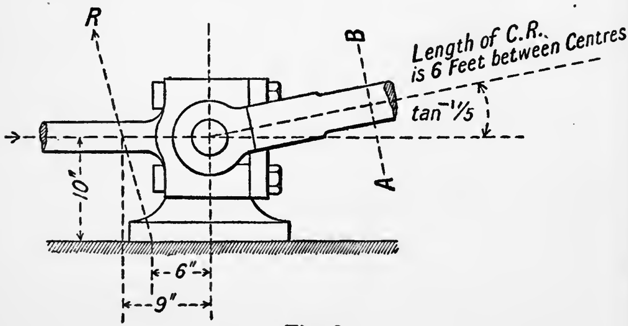


Fig. 8

ANSWERS.

PAPER No. 1.

(1) Reaction at B, horizontal component 770 lbs., vertical component 860 lbs.

(2) (*a*) 125 lbs. (*b*) 250 lbs.

(3) (*a*) 8·88 inches per sec. (*b*) 34 inches per sec. (*c*) 22·26 inch lbs. For reversed motion, couple given to shaft is 6·02 inch lbs.

(4) Distance traversed is 1260 feet.

(6) (*a*) 2·01 tons per sq. in. (*b*) 0·88 tons per sq. in. (*c*) 2·07 and $-0·21$ tons per sq. in.

(7) (*a*) 0·381 inch. (*b*) 0·254 inch.

PAPER No. 2.

(1) (*a*) 3·732. (*b*) 2·54. (*c*) 5600 lbs. and 3810 lbs.

(2) Speed acquired is 65·5 feet per sec.

(3) Percentage fluctuation is 0·43 on either side of the mean.

(4) Maximum compression is 7·3 inches.

(5) $E = 30{\cdot}9 \times 10^6$ lbs. per sq. inch, $C = 11{\cdot}6 \times 10^6$ lbs. per sq. inch, $m = 2{\cdot}96$.

(6) Greatest compression stress is 11·43 tons per sq. in., greatest tensile stress is 11·17 tons per sq. in.

(7) (*a*) 31800 lbs. per. sq. in. (*b*) 794 lbs. per sq. in. (*c*) Variation is sinusoidal.

(8) Horizontal thrust is 25 tons.

PAPER No. 3.

(1) 90 feet per sec.

(2) 77·5 lbs.

(3) (*a*) $\cot(\alpha + \phi)$. (*b*) $\dfrac{\tan \alpha}{\tan(\alpha + \phi)}$.

(5) Longitudinal stress is 2·2 tons per sq. in.

(6) Greatest bending moment is 25000 lbs. ft. Tension in the rod is 25000 lbs.

(7) 8570 lbs. per sq. in., longitudinal stretch is ·000495.

(8) (*a*) 141·4 tons feet. (*b*) 6·4 minutes.

PAPER No. 4.

(2) 0·946 lbs.

(3) Speed will lie between 104·7 and 124 revs. per minute.

(4) Time of a complete oscillation is 1 second.

(5) Limiting radius of curvature is 400 inches. Terminal couple is 3680 lbs. inches.

(6) Shear stress is 9780 lbs. per sq. inch. Axial elongation is 9·216 inches.

(7) Greatest bending moment occurs at a section distant 8′·8″ from one end. Its value is 112·7 ton feet.

(8) Principal stresses are + 1537 and − 264 lbs. per sq. inch.

PAPER No. 5.

(2) The 20 lbs. weight makes 108 oscillations per min. The 30 lbs. weight makes 88 oscillations per min.

(4) The girder makes 8·2 complete oscillations per sec.

(7) Vertical displacement of B is 0·227 inch. Horizontal displacement of B is 0·107 inch.

(8) Greatest longitudinal tension is 2·42 tons per sq. in. Least longitudinal tension is 2·00 tons per sq. in.

PAPER No. 6.

(2) 632 lbs.
(3) Centre of pressure is $\frac{1}{4}$ inch below centre of plate.
(4) Angular velocity is 14 radians per sec. or 2·23 revs. per sec.
(5) Least diameter is 0·2 inch.
(6) 13460 lbs. per sq. inch.
(7) Instability occurs when the load reaches the value $\frac{\pi^2}{4l^2} EI$.
(8) (*a*) 1800 lbs. per ft. run. (*b*) 28800 lbs. per ft. run.

PAPER No. 7.

(1) Force required is 5·23 tons.
(2) Speed is 6 feet per sec. Loss of kinetic energy is 140·6 inch tons. Tensile stress is 13·5 tons per sq. inch.
(4) Difference of water level is 10 inches.
(5) Greatest tensile stress is 10·36 tons per sq. in. Greatest compression stress is 11·30 tons per sq. inch.
(6) Decrease in volume is 0·81 cu. inch.
(7) For longitudinal seam, tensile stress is 10000 lbs. per sq. inch. For circumferential seam, tensile stress is 5000 lbs. per sq. inch. Maximum stress difference is 19750 lbs. per sq. inch. Maximum stretch is ·000603.
(8) Force required is 23 lbs.

PAPER No. 8.

(2) Drag is 6225 lbs.
(3) Retarding couple is 8·02 ft. lbs.
(4) Speeds at B, C, D are 120·5, 120·05, 120·37 revs. per min. respectively.
(5) Reaction at A is 12·2 tons inclined at 43° to the horizontal. Tension in tie rod is 6·4 tons.
(6) Vertical displacement of C is 0·100 inch. Horizontal displacement of C is 0·019 inch.
(7) Tensile stress is 9600 lbs. per sq. inch. Increase in diameter is 0·01 inch.
(8) Initial sag is 0·1035 inch.

PAPER No. 9.

(1) Reactions are 4·08, 6·44, 4·48 tons.

(2) 24·4 lbs.

(3) Revs. are 854 and 350 per min. Loss of kinetic energy is 2063 ft. lbs.

(4) $P = W$.

(6) Web carries 96·5 per cent. of the shearing force. Flanges carry 86·7 per cent. of the bending moment.

(7) Pull in the bolt is 6·3 tons.

PAPER No. 10.

(2) Pressures are 833·7 and 566·3 lbs. per sq. inch.

(3) The C.G. rises ·466 inch.

(4) Velocity acquired is 10·7 feet per sec.

(6) Tension in AE is 8·25 tons. Tension in BD is 9·5 tons.

(7) For upper plank greatest fibre stress is 500 lbs. per square inch.
For lower " " " " " 1000 " " " "

(8) Limiting pitch is 7·2 inches.

PAPER No. 11.

(1) Couple is 10·6 lbs. feet.

(2) Time of a complete oscillation is $\dfrac{2\pi}{\sqrt{\dfrac{12g}{dw^2} - 1}}$ secs.

(3) $a = \cdot 024$. Distances run are 321 feet and 49 feet. Steady speed 19·7 miles per hour.

(4) 203 secs.

(6) The two middle panels require counterbalancing.

(7) Longitudinal tensile stress at P is $\cdot 96M$ ton per square inch.

(8) Limiting load is 164300 lbs.

PAPER No. 12.

(2) Retardation is 4·73 feet per sec. per sec. Additional retardation due to axle friction is 0·161 feet per sec. per sec.

(3) Thrust in AB and AC is 95 tons. Tension in AD is 45·4 tons.

(4) 4·3 H.P.

(5) Longitudinal stress is 1750 lbs. per sq. inch. Hoop stress is 9720 lbs. per sq. inch.

(6) Reactions are 14 tons and 6 tons.

(7) Force required to draw the hoop is $31{\cdot}4\mu$ tons. Tyre should be made ·01725 inch too small.

PAPER No. 13.

(1) Velocity is 2 feet per sec. Loss of kinetic energy is 9 inch tons. Elongation of spring is 3 inches. Tension in wire is 3 tons.

(2) $2\frac{1}{2}$ inches.

(3) 554 and 831 revs. per minute. 1·45 secs.

(4) For maximum efficiency $\frac{u}{v} = 2$.

(5) According to maximum stress theory factor of safety is 2·00.
„ „ „ stretch „ „ „ „ 1·86.
„ „ „ slide „ „ „ „ 1·60.

(8) Thrust in AB is ·9 ton. Tension in BC is $\frac{1}{2}$ ton.
Horizontal reaction at A is $\frac{3}{2}$ tons. Vertical reaction at A is $2\frac{1}{2}$ tons.

PAPER No. 14.

(1) Velocity given to the plank is ·725 feet per sec. Time is 0·067 sec.

(2) Transverse metacentre is 4′·2″ above bottom. Righting moment is 325·5 lbs. feet.
Depths at two ends are 5·8 inches and 3·4 inches.

(4) Distance is $\frac{1}{8}$ inch. Relative velocity is 5·2 feet per sec.

(5) Residuary twisting couple is 23 lbs. feet.

(6) Vertical deflection is $\frac{1}{50}$ inch. Tension in each wire is increased by 2121 lbs.

(7) Terminal couples are 1333 inch lbs. and 666 inch lbs. Greatest shear stress is 6800 lbs. per sq. inch.

PAPER No. 15.

(1) $P = \frac{W}{4}$.

(3) 1000 revs. per minute.

(4) Mechanical advantage is 9. To lift 300 lbs. an effort of 44 lbs. is required.

Efficiencies are 55·5 %, 69·4 %, 75·8 %.

(5) Maximum tensile stress is 13440 lbs. per sq. inch.

(6) Shear stress is 0·625 tons per square inch.

(7) Couple required is $2 \cdot 0 \times 10^4 \theta$ lbs. inches.

(8) Limiting angular velocity is 7·36 radians per sec.

PAPER No. 16.

(1) (*a*) 452. (*b*) 392. (*c*) 0·27 inch.

(2) Pressure at crest is 7020 lbs. Pressure at hollow is 8020 lbs.

(4) Gyroscopic couple is 243 lbs. feet.

(5) Bending moment is $\frac{W}{2}[25 + 8 \tan \theta]$. It is least when $\theta = 0$.

(6) Tension in each spoke is 67 lbs.

(7) Thrust in each strut is 68 tons.

(8) Deflection in inches is $\frac{\omega^2}{6 \cdot 14 \times 10^6 - 10^2\omega^2}$.

Speed must not lie between 1900 and 3550 revs. per minute.

PAPER No. 17.

(1) Thrust is 320 lbs.

(2) Resultant pressure on the base acts at a part 5 feet from A measured along AB.

(5) Stress in BC is $10\sqrt{3}$ tons.

(6) Shear stress is 1·3 tons per square foot. Normal stress is $5\frac{3}{4}$ tons per square foot.

Stretch is $5 \cdot 2 \times 10^{-6}$.

(7) Sectional area of the steel is 0·648 inch. Bending moment is 109500 inch lbs.

PAPER No. 18.

(1) Angular velocity is $\frac{NB}{CB}\omega$.

(2) 0·74 lb.

(3) 1·3 inches.

(4) Critical engine speed is 342 revs. per minute.

(5) Energy lost per sec. in transmission is 3·2 ft. lbs. Strain energy dissipated per sec. is 2·4 ft. lbs.

(6) Couple required for rotation is 1400 inch lbs.

(7) 0·38 lb. Maximum stress is 1750 lbs. per sq. inch.

PAPER No. 19.

(1) Position of maximum normal pressure is 30° below the horizontal.

Braking couple is 171 ft. lbs.

(2) Piston will rise 2·2 inches. It finally comes to rest 1·6 inches above its initial position.

(3) 0·245 radian per sec.

(4) 14·85 feet per sec. per sec.

(6) For a length $\frac{W_1 l}{W_1 + W_2}$ the maximum B.M. is under the load W_1.

(7) (*a*) 500 lbs. (*b*) 0·12 inch. (*c*) 33 lbs. per square inch.

PAPER No. 20.

(1) 53 feet per sec. per sec.

(2) If the mass at time t is distant x inches above its mean position, the equation of motion is

$$\frac{d^4x}{dt^4} + 18g\frac{d^2x}{dt^2} + 64g^2x = 32ag\left[2g - \pi^2n^2\right]\cos 2\pi nt.$$

(3) Expenditure of energy is 1·30 ft. lbs. per sec.

(4) (*a*) 2180 lbs. (*b*) 7820 lbs. (*c*) 20380 lbs. (*d*) 1020 lbs.

(5) 313 tons.

(6) 74 tons.

(7) Greatest compression stress is 2·02 tons per square inch.

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